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THESIS

MISSILE TERMINAL GUIDANCE AND CONTROL AGAINST EVASIVE TARGETS

by

John CS. Swee

March 2000

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Second Reader:	Harold A. Titus

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Terminal guidance of the missile against an evasive target is explored. The two main types of guidance laws employed in the majority of missiles, namely proportional navigation and command to line-of-sight are modeled using Matlab® Simulink™. The two-dimensioned missile-target intercept geometry is simulated for a point mass missile and target. Missile velocity due to its drag also factored. The engagement results for different scenarios with target doing a 9-g evasive maneuver are then compared to analyze the performance of hybrid proportional navigation guidance with bang-bang control.

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MISSILE TERMINAL GUIDANCE AND CONTROL AGAINST EVASIVE TARGETS

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ABSTRACT

The ability of a missile to intercept a target in its flight is greatly determined by the guidance law employed in the guidance processing of the missile. Two main types of guidance laws are employed in the majority of missiles, namely proportional navigation (PN) and command to line-of-sight (CLOS). The effectiveness of CLOS however is limited to shorter ranges of up to about 6km, due to its sensitivity to angular tracking errors between the ground station and the target. PN is probably the most widely used homing guidance law, which seeks to null the line-of-sight (LOS) angle rate by making the missile turn rate be directly proportional to the LOS rate. PN does not suffer from the range limitation encountered by CLOS because it is self-homing and relies on an onboard seeker that provides target's LOS information directly. We modeled the two-dimensioned missile-target intercept geometry with CLOS and PN guidance laws using Matlab® Simulink™. The engagement results for a non-maneuvering target were first established as a benchmark and subsequently compared for the case of a target with a 9-g evasive maneuver. While conventional PN was shown to be effective against a non-maneuvering target, it has to be modified to improve its performance against a maneuvering target. Simulations for a proportional navigation strategy incorporating bang-bang control were carried out and analyzed. The performance of this strategy is also presented.

TABLE OF CONTENTS

I. INTRODUCTION

A. BACKGROUND

The sequence of launching a missile against an airborne target involves various stages. Of particular concern for this study is the terminal guidance of the missile to achieve target interception. The various methods of missile guidance can be broadly categorized under autonomous (i.e. self or homing) guidance, or ground commanded guidance.

Autonomous Guidance. This type of guidance requires the missile to carry its own guidance system. The major benefit of autonomous guidance is the ability of the missile to track its target after it is fired and frees the ground control station to perform other missions. This feature is also known as "fire and forget". The missile would need to carry a seeker, which invariably makes it more costly. Generally, proportional navigation is employed.

Ground Commanded Guidance. With ground commanded guidance, the computing power resides in a Fire Control System (FCS) at the launch platform. The FCS tracks both the target and missile until interception occurs. It computes the target trajectory, determines the necessary missile acceleration and transmits the guidance commands via an encoded up-link to the missile. This makes the missile less costly, but it is range limited, as the tracking error increases with target range from the FCS. Generally, ground commanded systems employs command to line-of-sight techniques.

B. OBJECTIVE

The ability of a missile to intercept a target in its flight is greatly determined by the guidance law employed in the guidance processing of the missile. The objective of this study is to modify the guidance law to improve its performance against a maneuvering target.

C. RELATED WORK

It is known that proportional navigation (PN) was optimal against a non-maneuvering target [Ref 1 & 3], but was not effective when the target maneuvers. Work in improving the missile guidance laws mostly began after WWII. The closed form solution for PN was only recently published in 1990 [Ref 7]. Efforts to modify the PN guidance had been carried out in recent decades, and new algorithms including augmented PN have been proposed to improve its performance against maneuvering target [Ref 10]. There are also efforts to evaluate the performance of missile guidance laws against a maneuvering target [Ref 8,9,10 & 11], but few work have been carried out with Bang-bang control together with PN.

D. THESIS ORGANIZATION

For this study, we will explore the performance of proportional navigation for a two-dimensioned intercept geometry of point mass target and missile.

Chapter II provides the reader with a basic understanding of the various guidance laws typically employed in today's arsenal of missiles.

Chapter III dwells on the problem formulation, employing proportional navigation for different target-missile intercept scenarios and geometries. We first assume a non-maneuvering target scenario and then repeat the scenario for a target that maneuvers with a 9-g turn about 2 seconds prior to the time of missile interception.

Chapter IV provides the details on the simulation model used to evaluate the performance of the modified proportional navigation guidance proposed in this thesis.

Chapter V presents the results and analysis of the simulations.

Finally, Chapter VI presents the conclusions and recommendations of this study.

II. MISSILE GUIDANCE LAWS

A. GENERAL

This chapter explains the various guidance laws that are typically implemented in the missile guidance processing. The missile guidance system provides the auto-pilot (i.e., missile control system) with the necessary lateral acceleration commands. The missile-target intercept geometry has several important parameters as shown Figure 1 below:

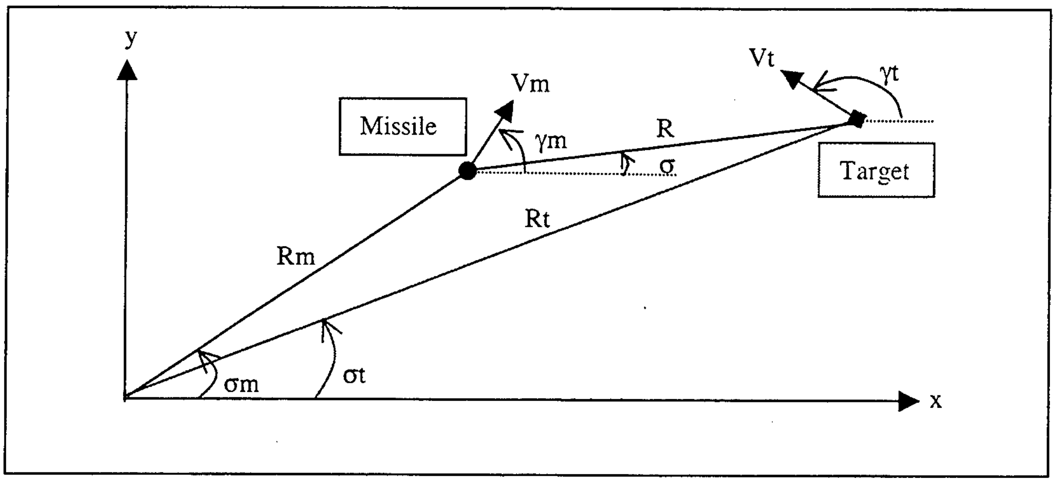

Figure 1. Missile-Target Intercept Geometry

Typical parameters that can be described in the missile-target intercept geometry are:

R_M,R_T: Tracker to missile range, Tracker to target range respectively

R : Range between target and missile

σ_m, σ_t: Tracker to missile angle, Tracker to target angle respectively

σ: LOS angle between missile and target

γ_m, γ_t: Missile velocity vector angle, Target velocity vector angle respectively

The following paragraphs will describe the major types of Command Guidance (i.e., Beam Rider and Command to line-of-Sight) and Homing Guidance (Pure Pursuit and Proportional Navigation).

B. BEAM RIDER GUIDANCE

Beam riding guidance is one of the simplest form of command guidance. The object of beam riding (BR) is to fly the missile along a tracker beam that is continuously pointed at the target. A typical BR geometry is shown in Figure 2 below.

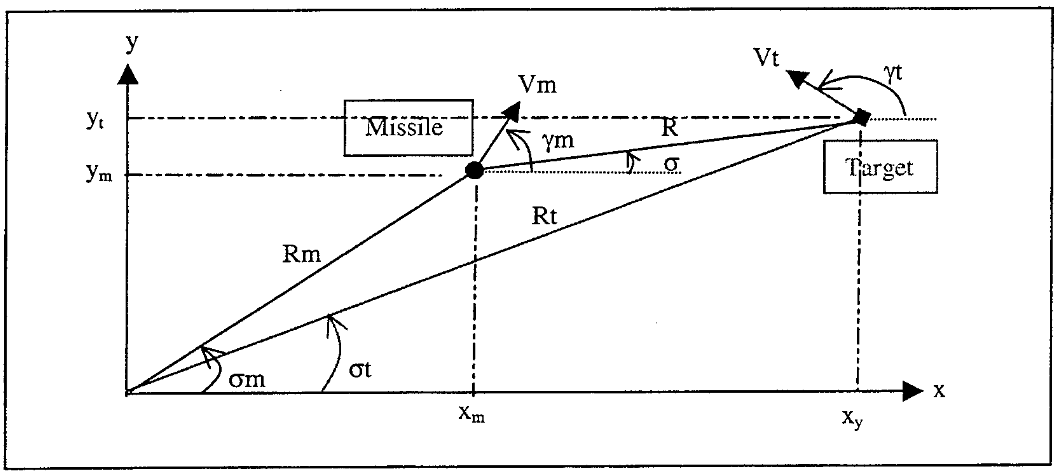

Figure 2. Missile-Target Intercept Geometry

From Figure 2, we obtain the various parameters as follows:

$$\sigma m = \tan^{-1}\left(\frac{y_m}{x_m}\right) \qquad \text{..........................} \qquad (2.1a)$$

$$\sigma t = \tan^{-1}\left(\frac{y_t}{x_t}\right) \qquad \text{..........................} \qquad (2.1b)$$

$$Rm = \sqrt{x_m^{\ 2} + y_m^{\ 2}} \qquad \text{..........................} \qquad (2.1c)$$

$$Rt = \sqrt{x_t^{\ 2} + y_t^{\ 2}} \qquad \text{..........................} \qquad (2.1d)$$

The crossing range error (CRE) i.e., the distance if the missile from the beam is:

$$CRE = Rm \sin(\sigma t - \sigma m) \qquad \text{..........................} \qquad (2.2)$$

If the missile is always on the beam (i.e., CRE=0), then the missile will surely hit the target. Hence, the BR guidance law to drive the miss distance to zero is to make the missile acceleration command n_c proportional to the CRE.

$$\begin{aligned} n_c &= K.CRE \\ &= K.Rm \sin(\sigma t - \sigma m) && \text{.....} \quad (2.3) \\ &\cong K.Rm.(\sigma t - \sigma m) && \text{for small } (\sigma t - \sigma m) \end{aligned}$$

Hence, we observe that the guidance command is proportional to the angular error between the missile position and the tracker beam. However, implementing the above BR guidance will result in an oscillatory missile acceleration (Ref [1]). A larger value of K (i.e. K=10) will give a smaller miss-distance but the missile acceleration oscillations increases. Conversely, a smaller value of K (i.e., K=1) will have less oscillation in its missile acceleration but a larger miss-distance.

In order to stabilize the BR guidance loop, we can add a lead-lag compensation network, such as (Ref [1]):

$$G(s) = K'\left(\frac{1 + s/2}{1 + s/20}\right) = 10K'\left(\frac{s+2}{s+20}\right) = K\left(\frac{s+2}{s+20}\right) \text{......} \quad (2.3)$$

C. COMMAND TO LINE-OF-SIGHT GUIDANCE

Beam riding guidance can be significantly improved by taking the beam motion into account. Adding the beam acceleration to the BR guidance equation yields command to line-of-sight.

Command to line-of-sight (CLOS) guidance keeps the missile in the LOS between the launch point and the target. A typical flight trajectory of a CLOS missile is shown in Fig. 3. The distance between the missile and the desired trajectory (i.e., the radar-to-target LOS line) is defined as the cross-range error (CRE). The ground control station will compute and provide necessary missile acceleration commands to bring the CRE to zero.

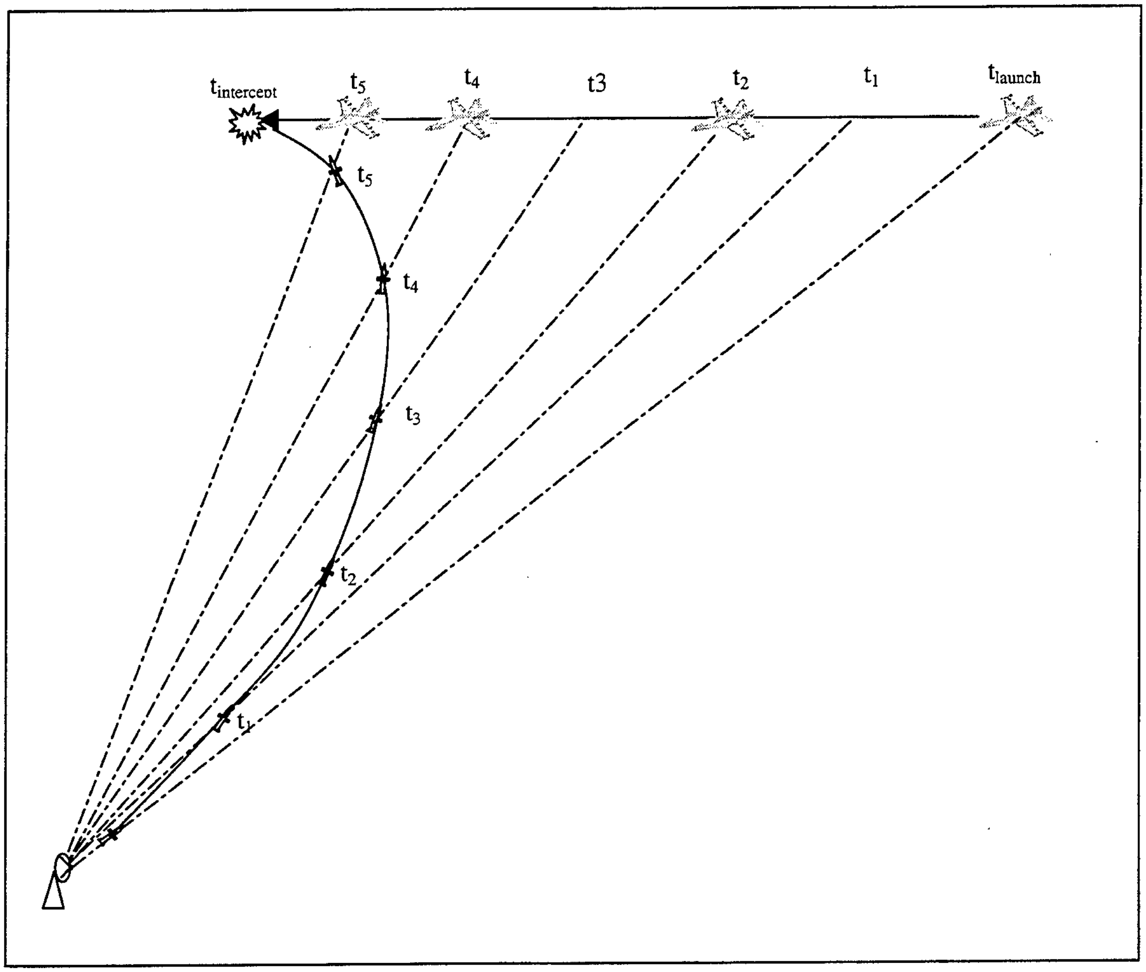

Figure 3. CLOS missile flight trajectory

The amount of error in the CRE at the point of intercept is dependent on the range from the launch point. This inherent disadvantage of CLOS restricts its use to short ranges.

The angle of the beam with respect to the target (see Fig. 2) is given by:

$$\sigma t = \tan^{-1}\left(\frac{y_t}{x_t}\right) \quad \text{..........................} \quad (2.4a)$$

Then the angular velocity and acceleration of the beam are:

$$\text{Angular Velocity :} \quad \dot{\sigma t} = \frac{d\left[\tan^{-1}\left(\frac{y_t}{x_t}\right)\right]}{dt}$$

$$= \frac{x_t \dot{y}_t - y_t \dot{x}_t}{x_t^{\,2} + y_t^{\,2}} \quad \text{..........} \quad (2.4b)$$

$$= \frac{x_t \dot{y}_t - y_t \dot{x}_t}{R_t^{\,2}}$$

$$\text{Angular Acceleration :} \, \ddot{\sigma t} = \frac{d\dot{\sigma t}}{dt}$$

$$= \frac{a_{t_y} \cos \sigma t - a_{t_x} \sin \sigma t - 2\dot{\sigma t}\dot{R}_t}{R_t}$$

$$where \;\; \dot{R}_t = \frac{x_t \dot{x}_t + y_t \dot{y}_t}{R_t} \quad \text{......} \quad (2.4c)$$

a_{t_x} and a_{t_y} are acceleration in x and y directions

The acceleration perpendicular to the beam a_{tp}, can be expressed in terms of the inertial coordinates of the target acceleration:

$$\begin{aligned} a_{t_p} &= -a_{t_x} \sin \sigma t + a_{t_y} \cos \sigma t \\ &= R_t \ddot{\sigma t} + 2\dot{R}_t \dot{\sigma t} \end{aligned} \quad \text{......} \quad (2.5)$$

In order for the missile to stay on the beam, we are striving to ensure that:

$$\begin{aligned} \dot{\sigma}_m &= \dot{\sigma t} \quad \text{and} \\ \ddot{\sigma}_m &= \ddot{\sigma t} \end{aligned} \quad \text{......} \quad (2.6)$$

Hence, the commanded missile acceleration perpendicular to the beam should be:

$$\begin{aligned} a_{m_p} &= R_m \ddot{\sigma} m + 2\dot{R}_m \dot{\sigma} m \\ &= R_m \ddot{\sigma} t + 2\dot{R}_m \dot{\sigma} t \end{aligned} \qquad \text{...... (2.7)}$$

Adding the beam acceleration term , a_{m_p} to the nominal acceleration generated by the beam rider equations yields the CLOS guidance. Hence for CLOS, the missile acceleration command is given by:

$$\begin{aligned} n_c &= K.CRE + a_{m_p} \\ &= K.Rm\sin(\sigma t - \sigma m) + R_m \ddot{\sigma} t + 2\dot{R}_m \dot{\sigma} t \qquad \text{.... (2.8)} \\ &\cong K.Rm.(\sigma t - \sigma m) + R_m \ddot{\sigma} t + 2\dot{R}_m \dot{\sigma} t \qquad \text{for small } (\sigma t - \sigma m) \end{aligned}$$

D. PURE PURSUIT

Pure Pursuit (PP) guidance seeks to keep the missile's velocity vector pointing at the target. Typically, it has relatively simple processing avionics and works well only against very slow targets. Against a fast moving crossing target, the lateral acceleration required from the missile at intercept tends to be very high.

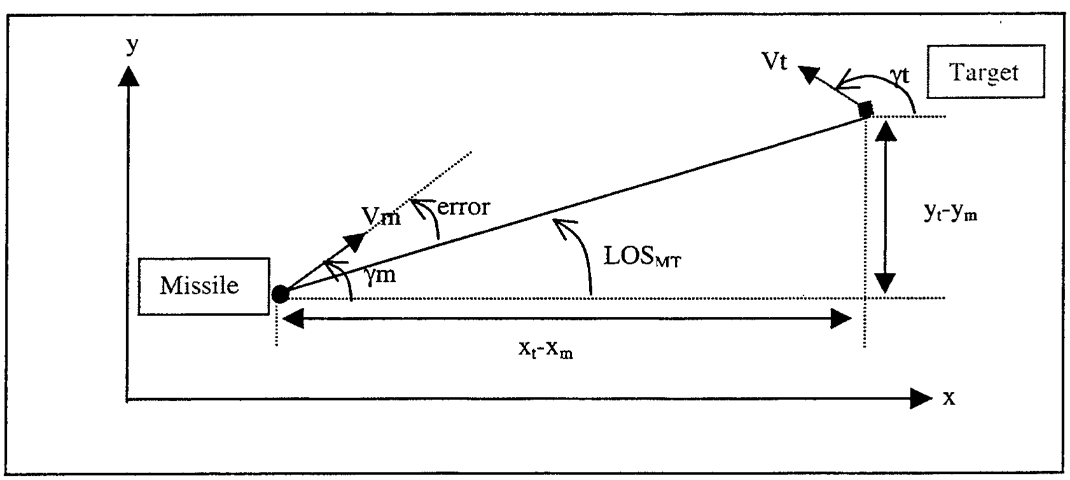

Figure 4. Pure Pursuit Geometry

Figure 4 shows PP target-missile intercept geometry. The angular error between the missile vector and the LOS of the target to missile (LOS_{MT}) is shown in the figure. The guidance algorithm computes the missile heading to bring the angular error to zero. PP guidance is not effective against a fast target, especially when the target maneuvers. Hence, it will not be considered in this study.

E. PROPORTIONAL NAVIGATION

While CLOS is typically employed for shorter engagement ranges; for longer ranges, Proportional Navigation (PN) guidance is preferred and some form of seeker (active or passive) is built into the missile to track the target.

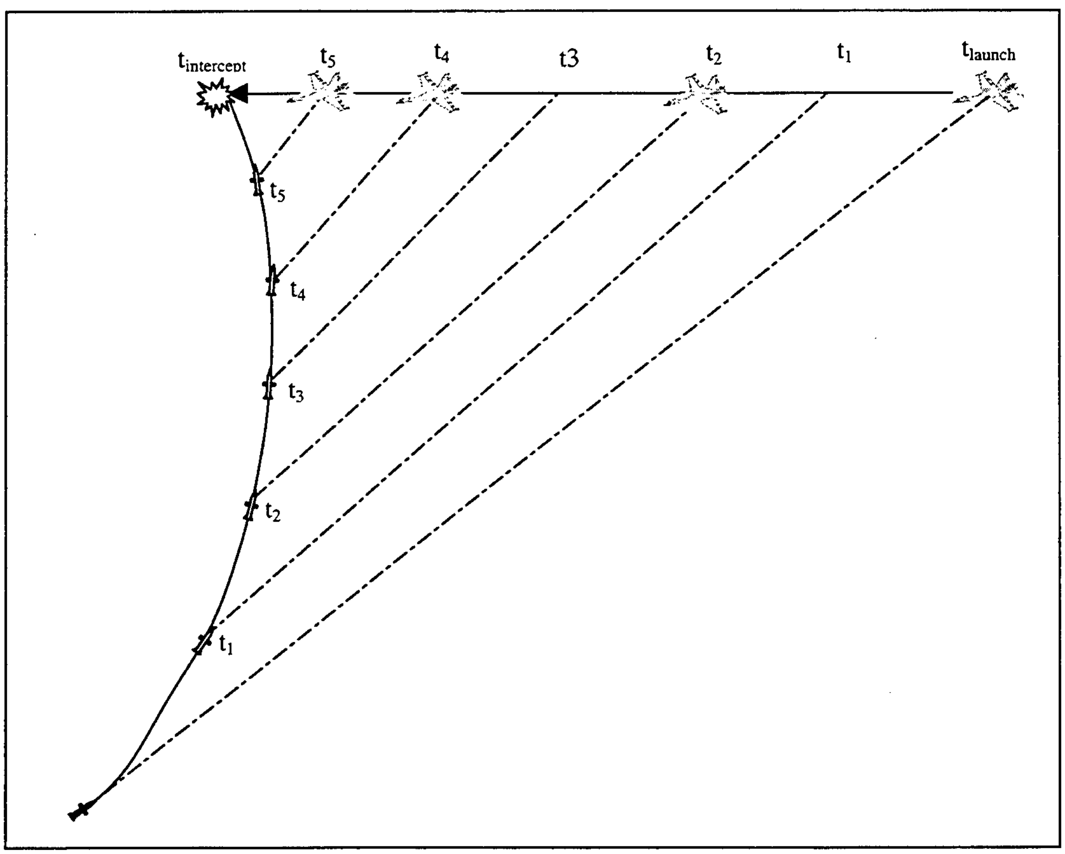

Figure 5. Proportional Navigation Flight Trajectory

PN guidance provides acceleration commands to the missile, which are proportional to the rate of change of the LOS_{TM}, i.e.:

$$\text{Commanded acceleration, } n_c = N'V_C\dot{\theta}_L$$

The PN class consists mainly of two kinds of guidance laws [Ref 8]:

a) For True Proportional Navigation (TPN), n_c is applied normal to the LOS. For TPN, the effective acceleration component normal to the missile vector is given as:

$$\Rightarrow \text{Commanded acceleration to missile, } a_m = \frac{n_C}{\cos(\theta_M - \theta_L)} \quad \ldots\ldots \quad (2.9)$$

b) For Practical/Pure Proportional Navigation (PPN), n_c is applied normal to the missile velocity vector, V_m.

$$\Rightarrow \text{Commanded acceleration to missile, } a_m = n_C = N'V_C\dot{\theta}_L \quad \ldots\ldots \quad (2.10)$$

F. BANG-BANG

For bang-bang control, instead of providing an acceleration command based on a proportionality relationship, the missile maneuvers by 'banging' to its designed g-limit to bring the missile along the LOS.

Thus the commanded acceleration based on the 'bang-bang' algorithm is:

$$\text{Commanded acceleration, } n_c = g_{\text{limit}} * 9.81 * sign(V_C\dot{\theta}_L) \quad \ldots\ldots \quad (2.11)$$

III. PROBLEM FORMULATION

A. MISSILE TARGET SCENARIO AND GEOMETRY

For this study, we examine the performance of proportional navigation for a two-dimensioned intercept geometry of point mass target and missile. The target and missile are assumed to be point mass models in a plane, moving with velocities V_T and V_M respectively.

The initial position of the missile is assumed to be the reference point of the relative coordinate system with its initial velocity vector pointing at the initial target position.

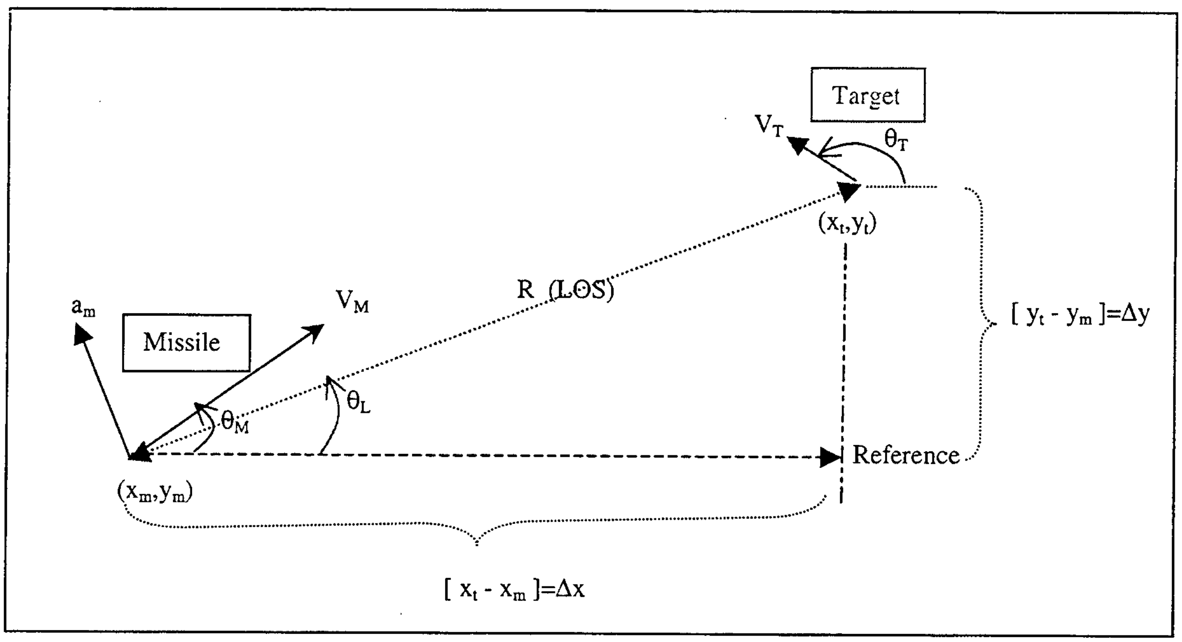

Figure 6. Proportional Navigation Geometry

From Figure 6, we obtain the various parameters as follows:

Angular components:

$$\theta_L = tan^{-1}\left(\frac{y_t - y_m}{x_t - x_m}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.1a)$$

$$\theta_M = \tan^{-1}\left(\frac{\dot{y}_m}{\dot{x}_m}\right) \qquad \text{........................} \qquad (3.1b)$$

$$\theta_T = \tan^{-1}\left(\frac{\dot{y}_t}{\dot{x}_t}\right) \qquad \text{........................} \qquad (3.1c)$$

Closing Velocity:

$$V_c = -\frac{dR}{dt} = -V_T \cos(\theta_T - \theta_L) + V_M \cos(\theta_M - \theta_L) \qquad \text{........} \qquad (3.1d)$$

Alternatively, using pythagoras theorem:

$$R^2 = \Delta x^2 + \Delta y^2$$

$$\frac{dR^2}{dt} \Rightarrow 2R\frac{dR}{dt} = 2(\Delta x)\Delta\dot{x} + 2(\Delta y)\Delta\dot{y}$$

$$\Rightarrow \frac{dR}{dt} = \frac{(\Delta x)\Delta\dot{x} + (\Delta y)\Delta\dot{y}}{R}$$

Since $\Delta x = R\cos(\theta_L)$

$\Delta y = R\sin(\theta_L)$

we have:

$$V_c = -\frac{dR}{dt} = -[\Delta\dot{x}\cos(\theta_L) + \Delta\dot{y}\sin(\theta_L)] \qquad \text{.................} \qquad (3.1e)$$

LOS rate:

$$\dot{\theta}_L = \frac{d\theta_L}{dt} = \frac{d\left[\tan^{-1}\left(\frac{y_t - y_m}{x_t - x_m}\right)\right]}{dt} \qquad \text{.................} \qquad (3.1f)$$

$$= \frac{\Delta\dot{y}\cos(\theta_L) - \Delta\dot{x}\sin(\theta_L)}{R}$$

B. TARGET AND MISSILE MANUEVER

Turn rate of an aircraft is a function of it's speed and acceleration (Ref [1]). The maneuver by target and missile in this study is implemented by using turn rates. Fig. 6 shows an aircraft moving at a constant speed along a circle of radius R.

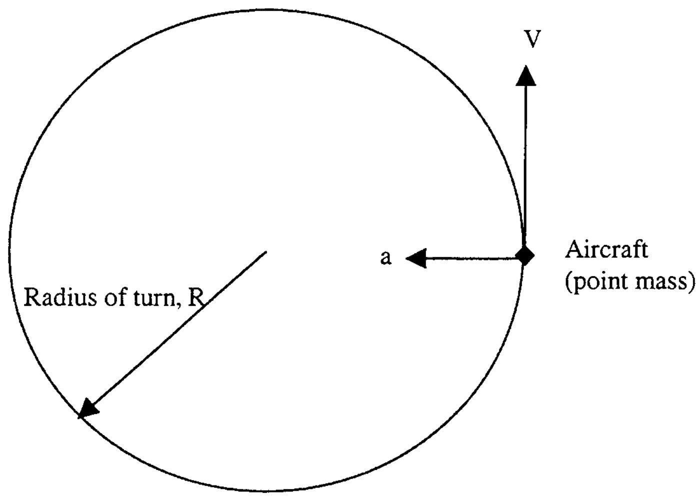

Figure 7. Aircraft (point mass) during a turn

For short flight time, the coriolis effect can be neglected. The relationship of the instantaneous velocity V, and the instantaneous acceleration a, is: $\|a\| = \dfrac{\|\mathbf{v}\|^2}{R}$

Then, the turn rate, $$\varpi = \frac{2\pi}{2\pi R / \|\mathbf{v}\|} = \frac{\|\mathbf{v}\|}{R} = \frac{\|\mathbf{a}\|}{\|\mathbf{v}\|} \quad \text{........................} \quad (3.2)$$

The acceleration producing the turn is perpendicular to the velocity and can be represented as (Ref [5]):

$$\mathbf{a} = \begin{bmatrix} \dot{V}x \\ \dot{V}y \end{bmatrix} = \varpi\|\mathbf{v}\| \begin{bmatrix} \cos(90°) & -\sin(90°) \\ \sin(90°) & \cos(90°) \end{bmatrix} \frac{\mathbf{v}}{\|\mathbf{v}\|} = \begin{bmatrix} -\varpi Vy \\ \varpi Vx \end{bmatrix} \text{.........} \quad (3.3)$$

From this we obtained the continuous-time state equation as:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{V}x(t) \\ \dot{y}(t) \\ \dot{V}y \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\varpi \\ 0 & 0 & 0 & 1 \\ 0 & \varpi & 0 & 0 \end{bmatrix} \mathbf{x}(t) \qquad \text{...........} \qquad (3.4)$$

$$= A(\varpi)\mathbf{x}(t)$$

C. GUIDANCE LAW IMPLEMENTATION

In this study, we study the performance of proportional navigation with N'=3 in combination with bang-bang guidance, subject to a 20-g limit.

Proportional Navigation: The proportional navigation guidance Law is given by:

$$\text{Commanded acceleration, } n_c = N' V_C \dot{\theta}_L$$

a) For True Proportional Navigation (TPN), n_c is applied normal to the LOS.

$$\Rightarrow \text{Commanded acceleration to missile, } a_m = \frac{n_c}{\cos(\theta_M - \theta_L)}$$

b) For Practical/Pure Proportional Navigation (PPN), n_c is applied normal to the missile velocity vector, V_m.

$$\Rightarrow \text{Commanded acceleration to missile, } a_m = n_c$$

$$\text{We know that turn rate, } \varpi = \frac{\text{acceleration}_{\text{perpendicular}}}{\|\text{Velocity}\|} \Rightarrow \varpi_m = \frac{a_m}{\|\mathbf{v}_m\|}$$

Thus the corresponding commanded turn-rate is given by:

a) For TPN: $$\varpi_m = \frac{a_m}{\|\mathbf{v}_m\|} = \frac{N' V_c \dot{\theta}_L}{V_M \cos(\theta_M - \theta_L)} \qquad \text{...........} \qquad (3.5)$$

b) For PPN: $$\varpi_m = \frac{a_m}{\|\mathbf{v}_m\|} = \frac{N' V_c \dot{\theta}_L}{V_M} \qquad \text{...........} \qquad (3.6)$$

where:

N' is the proportional constant, which typically varies from 3-5.

V_c is the closing velocity obtained from Eqn 3.1(d) or 3.1(e)

$\dot{\theta}_L$ is LOS rate obtained from Eqn 3.1(f)

V_M is the instantaneous missile speed

Bang-Bang: For Bang-bang control, instead of a proportionality relationship, a full 20-g acceleration in either direction of the LOS is applied to null the LOS rate [Ref 6]. This is implemented by: Commanded acceleration, $n_c = 20 * 9.81 * sign(V_C \dot{\theta}_L)$

Guidance Laws' Implementation. This study examines the following schemes of guidance laws:

a) TPN guidance alone

b) Bang-bang guidance alone

c) Hybrid TPN and Bang-bang guidance

   i) Start with Bang-bang and switches to TPN at R<2km

   ii) Start with TPN and switches to Bang-bang at R<2km

D. TURN-RATE TIME CONSTANTS

A parameter associated with missile (as well as target) maneuverability is the turn-rate time constant, T_{turn}. The relationship of the turn-rate time constant is given as :

$$\begin{aligned} \varpi_{output} &= \frac{n_c}{|V|}(1 + T_{turn} s) \\ &= \varpi_c (1 + T_{turn} s) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.7) \end{aligned}$$

Generally, T_{turn} increases with altitude and decreases with missile velocity. A fast turn-rate time constant with a large navigation constant may result in instability in the overall guidance system [Ref 1]. Typical time constant varies from 0.5-1.0 seconds.

E. MISSILE DRAG

The missile speed, V_M, will encounter atmospheric frictional drag during its flight. This will decrease its speed during the coast phase of its flight. The atmospheric drag is computed as follows [Ref 1]:

$$\text{Drag} = \rho * C_D * \text{Area} * V_M^2/2 \qquad (3.8)$$

where:

V_M = Missile speed

Area = Reference area

(Which can be taken to be π*radius^2 for a cruciform missile body.

We assume a medium-sized 300 kg missile with a radius=0.15m)

C_D = Coefficient of Drag

($C_D = 0.2$ was chosen for supersonic speed during its coasting flight)

ρ = Atmospheric density

i) For height < 9144m, $\rho=1.22557*\exp(-h/9144)$

ii) For height > 9144m, $\rho=1.75228763*\exp(-h/6705.6)$

Missile engagement against an aircraft typically occurs below 9144m. For this simulation, an arbitrary height of 2000m was used.

The drag expression given in Equation 3.8 applies to a frontal atmospheric drag. We can expect the drag to increase as the missile does a turn, since the effective area encountering

atmospheric friction in the direction of the missile velocity vector increases. The coefficient of drag, C_D, increases as a function of coefficient of lift C_L. The relationship is (Ref [2] & [9]):

$$C_D = C_{D0}\,(1+k_1\,C_L^2)$$

where :

C_{D0} is the nominal coefficient of drag with angle of attack is zero.

k_1 = a constant unique to the aerodynamic surfaces of the missile

C_L = coefficient of lift for corresponding angle of attack

We attempt to establish a relationship of turn-rate on the missile drag. We assume that when the missile is doing its maximum turn-rate at 20-g, the drag coefficient, C_D, increases by a factor of 5. Following the above equation, a similar relationship of C_D due to missile turn rate can be assumed to be described by:

$$C_D = C_{D0}\,(1+k_2\,\varpi^2) \qquad \text{..............................} \qquad (3.9)$$

where :

ϖ = missile turn-rate

$k_2 = 4/\varpi^2_{max}$, assuming that at maximum turn-rate, $C_D = 5C_{D0}$

F. MISSILE VELOCITY COMPENSATION

Since the missile is expected to encounter deceleration, which causes the missile speed V_m to change, it is reasonable that some improvement might be gained by compensating for the velocity change (Ref [4]). The conventional TPN guidance is modified as follows:

$$a_m = N'V_c\dot{\theta}_L - \dot{V}_M \sin(\theta_M - \theta_L) \qquad \text{.............} \qquad (3.10)$$

G. SCENARIOS OF MISSILE-TARGET INTERCEPTION

The scenario adopted in this study assumes an inbound target flying about 500m/s (i.e. about Mach 1.5). The missile is assumed to coast at an initial speed that is twice the target's speed, i.e. 1000m/s. The initial positions of the missile and target were selected so that the interception range is about 6-6.5km.

The following scenarios used to evaluate the performance of the Guidance scheme are:

<u>Scenario #1</u>: Crossing target with lateral distance of 1000m (See Fig. 8).

Part A: Target on straight course

Part B: Target does a 9-g turn away from missile at about 2 seconds prior to interception

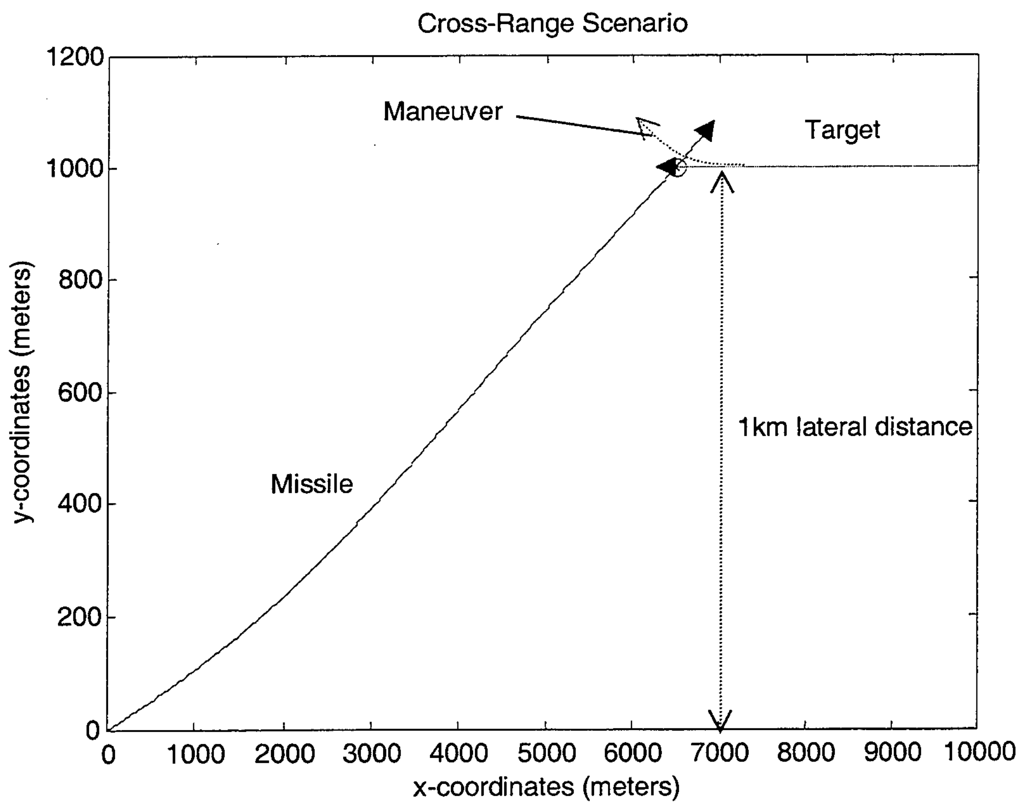

Figure 8. Scenario #1: Crossing Target with lateral distance of 1km

Scenario #2: Crossing Target at 45°. (See Fig. 9).

Part A: Target on straight course

Part B: Target does a 9-g turn away from missile at about 2 seconds prior to interception

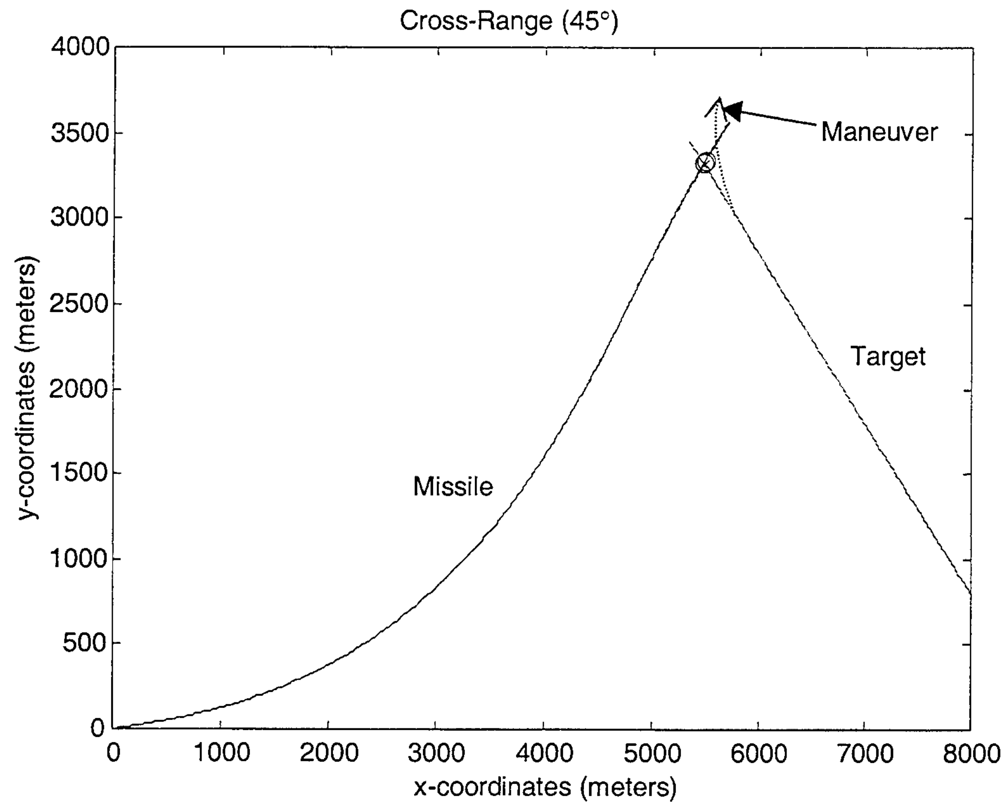

Figure 9. Scenario #2: 45° Crossing Target

Target Initialization:

	Scenario#1: Lateral Cross-Range	**Scenario#2:** 45° Cross-Range
x_t	10000	8000
Vx_t	-500	-500sin(45°)
y_t	1000	800
Vy_t	0	500sin(45°)

Missile Initialization:

	Scenario#1: Lateral Cross-Range	**Scenario#2:** 45° Cross-Range
x_m	0	0
Vx_m	200	200
y_m	0	0
Vy_m	500	500

H. NOISE IN LOS RATE

The key parameter in the proportional navigation guidance law is the LOS rate. The performance of the different guidance schemes is first evaluated without noise in the system. Subsequently, a sensitivity analysis was carried out to evaluate the performance of the proposed guidance strategy by introducing an additive white gaussian noise in the LOS rate.

The performance of the guidance strategy was evaluated for a range of standard deviations in the noisy LOS rate from 0.001 to 0.05 radians/sec (0.06 to 2.86 deg/s).

IV. SIMULATION

A. OVERVIEW OF MODEL

The guidance system simulation model is generated using MATLAB® and SIMULINK™. It comprises of the following blocks:

1. Initialization
2. Target Dynamics
3. Missile Dynamics
4. Guidance

The above simulation blocks are described in the following paragraphs in this Chapter. The outputs of the simulation are target and missile state vectors, LOS rate, and missile acceleration commands.

B. INITIALIZATION

Before the simulation can be run, the initial target and missile state vectors, as well as the sampling time and duration of simulation must be known. This initialization is given in the Matlab file 'Init.m'. The following variables/vectors are initialized:

- tinit : target state vector for the respective scenario
- minit : missile state vector for the respective scenario
- sampling time
- Drag component, beta = $\rho * C_D * Area * /2$
- V_{launch}: closing speed measured at launch

C. TARGET DYNAMICS BLOCK

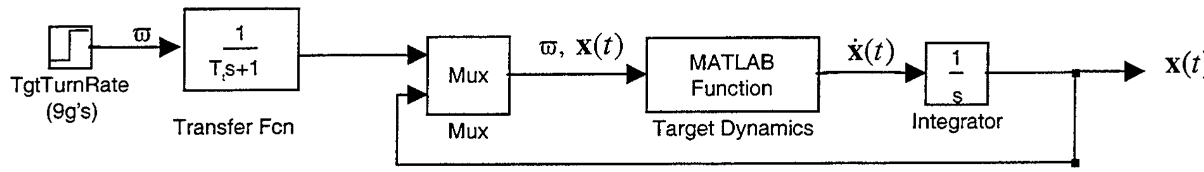

Figure 10. Target Dynamics Block

The turn-rate ϖ, is given as a step function with an amplitude equal to 9-g (i.e., 9*9.81 m/s^2). This is used to give the target a 9-g evasive maneuver at about 2 seconds from the expected time of intercept. It has a turn-rate time constant T_t= 1 second.

From Equation 3.4, the target state equation is:

$$\dot{\mathbf{x}}_{target}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{V}x(t) \\ \dot{y}(t) \\ \dot{V}y \end{bmatrix}_{target} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\varpi \\ 0 & 0 & 0 & 1 \\ 0 & \varpi & 0 & 0 \end{bmatrix} \mathbf{x}_{target}(t)$$
$$= A(\varpi)\mathbf{x}_{target}(t)$$

This is performed by the Matlab® function 'Targetdynamics.m', which receives ϖ and $\mathbf{x}(t)$ as inputs, and provides $\dot{\mathbf{x}}(t)$ based on Equation 3.4 as its output.

The integrator block is initialized to the target initial state vector 'tinit'.

D. MISSILE DYNAMICS BLOCK

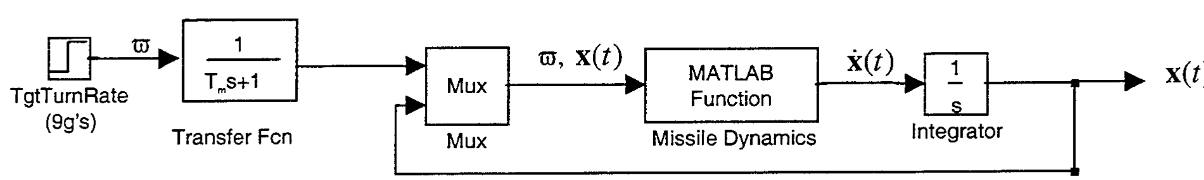

Figure 11. Missile Dynamics Block

The missile dynamics block is similar to the target dynamics block with a turn-rate time constant $T_m = 0.25$ to 1.0 seconds

The missile state vector is coded in the file 'Missiledynamics.m', which also includes the missile drag component.

Missile Drag. The missile drag component is implemented as follows:

a). Atmospheric drag is given as:

$$\text{Drag} = \text{beta} * V_m^2$$

where

beta=ρ*C_D*Ref_area /2 (initialized in 'Init.m')

Since Force=mass*acceleration, the deceleration due to drag, a_{drag} is:

$$a_{drag} = \text{Drag}/\text{mass}_{\text{missile}}$$

$$= \text{beta} * V_m^2/\text{mass}_{\text{missile}}$$

The corresponding turn-rate due to a_{drag} is:

$$\varpi_{drag} = a_{drag} / V_m$$

$$= \text{beta} * V_m/\text{mass}_{\text{missile}}$$

b). Additional drag due to missile turn-rate ϖ, is assumed to have a relationship:

$$k_2\, \varpi^2 \varpi_{drag}$$

where

$k_2 = 4/\varpi^2_{\max}$, assuming that at maximum turn-rate, the total drag component increases to 5 times the nominal drag.

E. GUIDANCE BLOCK

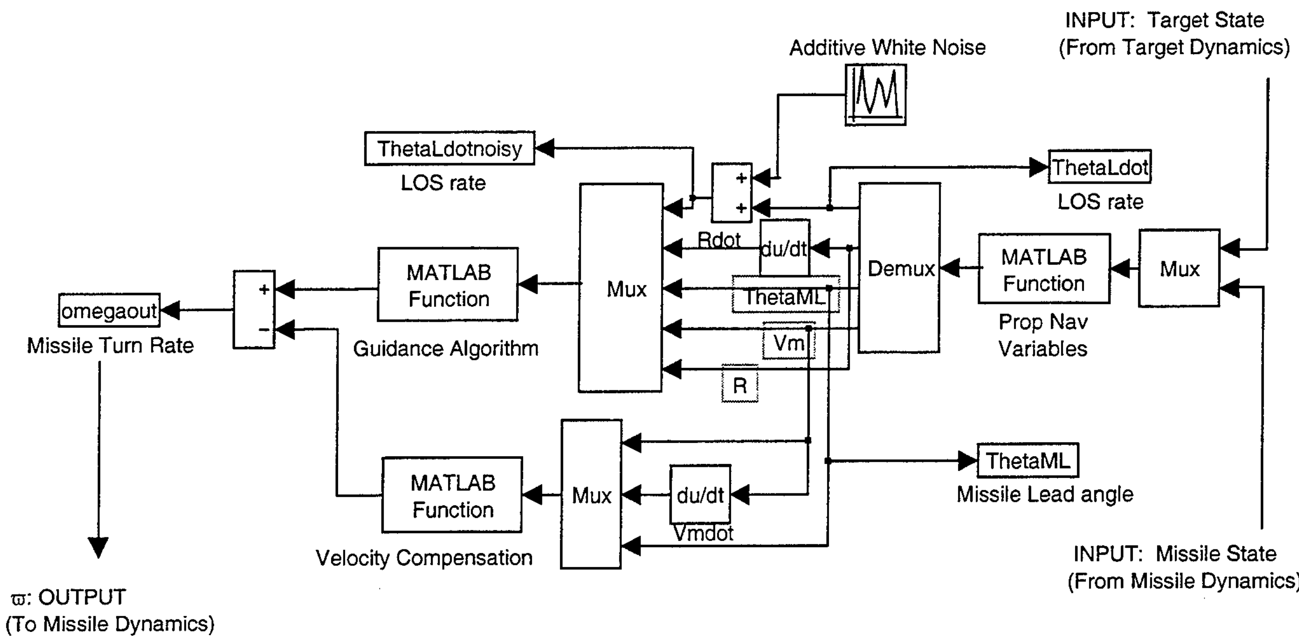

Figure 12. Missile Dynamics Block

The missile guidance block comprises the following:

1) Inputs: The input to the guidance block are the target and missile state vectors

2) Proportional navigation variables: The intercept geometry is obtained from the target and missile vectors. The Matlab® file 'PNvariables.m' calculates the respective variables used in proportional navigation as described in Equations 3.1(a) – 3.1(e).

3) Guidance Algorithm: The guidance law is coded in this block. The different guidance strategies are coded in the following Matlab® files:

   a. 'PNGuidance.m': Conventional proportional navigation with N'=3

   b. 'PNBangbang.m': 20-g bang-bang guidance

   c. 'BangPN2km.m': Starts with bang-bang and switches to proportional navigation when the range of target-to-missile is less than 2km

d. 'PNbang2km.m': Starts with Proportional navigation and switches to bang-bang when the range of target-to-missile is less than 2km

4) Velocity Compensation: An additional turn rate to compensate for the missile's velocity changes is coded in the Matlab® file 'Velcomp.m'.

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V. PERFORMANCE EVALUATION

A. PROPORTIONAL NAVIGATION

In an ideal case, where the missile is assumed to respond instantaneously to the guidance commands (i.e., turn-rate time constant, $T_m = T_t = 0$), conventional proportional navigation guidance (PNG) performs optimally for a non-maneuvering target. In practice, the missile, as well as the target, are expected to have turn-rate time constants. In addition, the missile which is not in sustained flight (i.e. propulsion is burnt), will have its speed reduced by aerodynamic drag.

However when turn-rate time constants are considered in the guidance system, PNG resulted in significant miss distances when the time constant is large and especially when the target does a 9-g turn away from the missile during the terminal phase. We assumed a typical turn-rate time constant $T_t = 1$ second, for the target and varied the missile turn-rate time constant, T_m, between 0.25 and 1 seconds.

The simulation was run for missile drag due to atmospheric friction alone (Tables 1(a)) and also for the case of additional drag due to the missile turn-rate (Table 1(b)). Detailed results of the simulations are given in Appendix C. A summary of the miss distances is tabulated below for the two scenarios considering different missile turn-rate time constants.

Time constant, T_m (sec)	**Miss Distance (m)**			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
0.25	0.5096	6.0796	0.3271	0.1018
0.5	0.4969	18.6431	0.2782	3.0831
1.0	0.6741	34.1685	6.8659	4.3064

Table 1a. Miss Distances for PNG ($T_t = 1.0$s and $T_m = 0.25$s, 0.5s, 1.0s) without considering drag due to missile turn-rate

Time constant, T_m (sec)	Miss Distance (m)			
	Scenario#1		Scenario#2	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
0.25	0.6468	7.5479	0.3601	1.2425
0.5	0.1290	19.8792	0.1381	4.5232
1.0	0.6207	34.7688	2.6993	5.3368

Table 1b. Miss Distances for PNG (T_t = 1.0s and T_m = 0.25s, 0.5s, 1.0s) with additional drag due to missile turn-rate

From the results, we observed that the missile time constant has a significant impact on the miss distance. A larger time constant makes the missile sluggish and less able to respond to the given guidance commands especially when the target maneuvers.

Scenario#1 presents a more difficult target to intercept than scenario#2, resulting in larger miss distances. This is because the geometry of scenario#1 requires the missile to perform a larger lateral acceleration during the terminal flight as it approaches the target.

We observed that T_m = 1.0s gave unacceptably large miss distances. Thus, the missile time constant should be less than 1 second for the above scenarios. For the subsequent simulations, we assume T_t = 1.0s and T_m = 0.5s and explore modifications to the guidance strategy to seek improvements.

B. VELOCITY COMPENSATED PROPORTIONAL NAVIGATION

The aerodynamic drag simulated in our model caused a reduction in missile velocity of about 150 m/s over 7 seconds of missile flight (see Fig. 11a), when we only consider frontal atmospheric drag.

With the additional drag due to missile turn-rate, the missile velocity profile is similar to Fig 11a when the target does not maneuver. However, the missile velocity reduces sharply when the missile turn-rate increases to respond to target maneuvers (see Fig 11b).

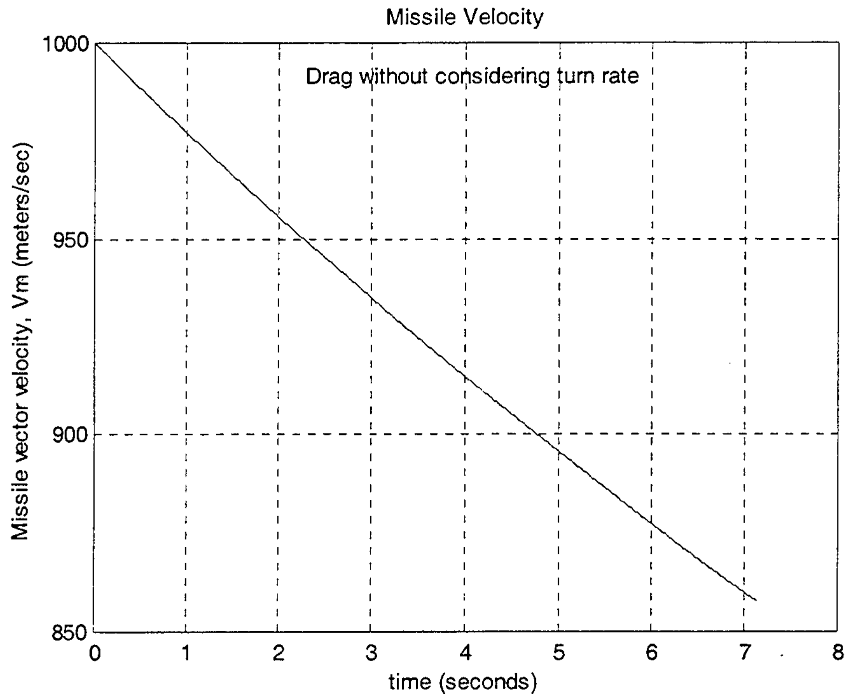

Figure 11a. Missile velocity for scenario#1 without considering drag due to turn-rate

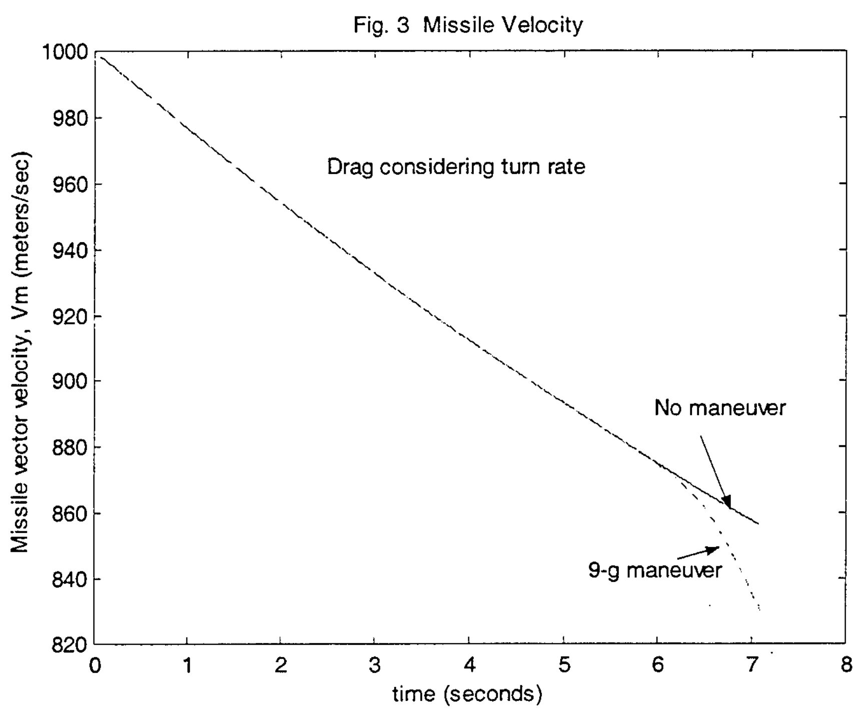

Figure 11b. Missile velocity for scenario#1 taking into consideration drag due to turn-rate

Velocity compensated proportional navigation (VCPN) is a modification of PNG to compensate for the change in missile velocity by an additional acceleration command $\dot{V}_m \sin(\theta_M - \theta_L)$.

Detailed results of the all the simulation runs in this study are given in Appendix C.

A summary of the miss distances is tabulated below for the two scenarios considering different drag model.

	Miss Distance (m)			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
PNG	0.4969	18.6431	0.2782	3.0831
VCPN	0.5420	18.1240	0.0605	2.8216

Table 2a. Miss Distances for VCPN guidance ($T_t = 1.0$s and $T_m = 0.5$s) without considering drag due to missile turn-rate

	Miss Distance (m)			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
PNG	0.1290	19.8792	0.1381	4.5232
VCPN	0.0480	19.0654	0.1240	3.9718

Table 2b. Miss Distances for VCPN guidance ($T_t = 1.0$s and $T_m = 0.5$s) with additional drag due to missile turn-rate

From the results, VCPN guidance does not offer much improvement. This is not surprising as the velocity compensation component, $\dot{V}_m \sin(\theta_M - \theta_L)$, is small, especially when the missile lead angle is often less than a few degrees. Furthermore, since any change in the missile velocity will be reflected in the closing velocity, Vc, the conventional PNG which computes its acceleration commands using Vc, in essence will address the change in missile velocity.

C. BANG-BANG GUIDANCE

For PNG, the responsiveness of the guidance against target maneuvers is improved with larger proportional navigation constant. This led to considering a bang-bang guidance strategy.

Detailed results of the simulations are given in Appendix C. A summary of the miss distances is tabulated below for the two scenarios considering different drag models.

	Miss Distance (m)			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
PNG	0.4969	18.6431	0.2782	3.0831
VCPN	0.5420	18.1240	0.0605	2.8216
Bang-bang	0.1116	0.3318	0.5761	0.5214

Table 3a. Miss Distances for Bang-bang guidance ($T_t = 1.0s$ and $T_m = 0.5s$) without considering drag due to missile turn-rate

	Miss Distance (m)			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
PNG	0.1290	19.8792	0.1381	4.5232
VCPN	0.0480	19.0654	0.1240	3.9718
Bang-bang	0.4433	0.5531	0.0359	0.0854

Table 3b. Miss Distances for PNG ($T_t = 1.0s$ and $T_m = 0.5s$) with additional drag due to missile turn-rate

The simulation results showed that the performance of Bang-bang guidance against a maneuvering target was much better than both conventional PNG and VCPN.

However, bang-bang is sensitive to changes in sign of LOS rate, which occurs when the guidance law tries to null the LOS rate. This makes bang-bang inherently more susceptible to noisy LOS rate.

In scenario#1, which presents more frequent sign changes in LOS rate about zero, we observed that the amount of control effort used by bang-bang was considerably higher at about 144-149 compared to 75-76 for the case of PNG and VCPN (see Table 4).

	Missile Control Effort (Integration of turn-rate)			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
PNG	76.2543	200.9011	622.9764	711.1141
VCPN	75.502	199.255	608.6091	681.0754
Bang-bang	149.3850	243.9412	671.2259	701.4282

Table 4a. Missile Control Effort (T_t = 1.0s and T_m = 0.5s) without considering drag due to missile turn-rate

	Missile Control Effort (Integration of turn-rate)			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
PNG	76.2628	196.771	651.076	736.5428
VCPN	75.4792	196.0564	609.9731	698.861
Bang-bang	144.6337	240.7891	655.8412	707.0795

Table 4b. Missile Control Effort (T_t = 1.0s and T_m = 0.5s) with additional drag due to missile turn-rate

Bang-bang has a disadvantage of frequent swinging of acceleration commands when LOS rate is near zero, requiring larger missile control efforts.

D. COMBINATION OF PROPORTIONAL NAVIGATION WITH BANG-BANG

It is conceivable that we can employ the advantage of proportional navigation guidance with Bang-bang. Hence a combined strategy of VCPN and bang-bang guidance was explored. Two strategies were examined:

a. Bang_PN: Begins with bang-bang and switches to proportional navigation when the missile approaches within 2km from the target.

b. PN_Bang: Begin with proportional navigation and then switches to bang-bang when the missile approaches within 2km from the target.

A summary of the miss distances is tabulated below for the two scenarios considering different drag models.

	Miss Distance (m)			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
PNG	0.4969	18.6431	0.2782	3.0831
VCPN	0.5420	18.1240	0.0605	2.8216
Bang-bang	0.1116	0.3318	0.5761	0.5214
Bang-PN	0.1118	14.7840	0.5185	3.8627
PN-bang	0.5422	0.5369	0.4403	0.3169

Table 5a. Comparison of Miss Distances ($T_t = 1.0s$ and $T_m = 0.5s$) without considering drag due to missile turn-rate

	Miss Distance (m)			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
PNG	0.1290	19.8792	0.1381	4.5232
VCPN	0.0480	19.0654	0.1240	3.9718
Bang-bang	0.4433	0.5531	0.0359	0.0854
Bang-PN	0.4482	14.9029	0.4061	7.2254
PN-bang	0.0512	0.7763	0.3128	0.2759

Table 5b. Comparison of Miss Distances ($T_t = 1.0s$ and $T_m = 0.5s$) with additional drag due to missile turn-rate

Bang-PN Strategy

We observed that the Bang-PN strategy produced the largest miss distances against maneuvering targets. Switching to PN guidance at the terminal stage when the target maneuvers is not effective. In fact, it is when the missile is nearer to the target, that larger accelerations are often required from the missile to achieve target interception. This strategy is thus not effective.

PN-Bang Strategy

On the other hand, we find that the miss distances obtained using the PN-Bang strategy against the 9-g target maneuvers were much lower than PNG, VCPN and Bang-PN, and comparable to Bang-bang's results.

A comparison of the amount of missile control effort in given in Table 6. We observe that the control effort required by the PN-Bang strategy is almost consistently the lowest.

	Missile Control Effort (Integration of turn-rate)			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
PNG	76.2543	200.9011	622.9764	711.1141
VCPN	75.502	199.255	608.6091	681.0754
Bang-bang	149.3850	243.9412	671.2259	701.4282
Bang_PN	145.5924	253.6009	678.3973	769.8148
PN_Bang	75.5047	193.2745	640.6799	667.8471

Table 6a. Missile Control Effort ($T_t = 1.0s$ and $T_m = 0.5s$) without considering drag due to missile turn-rate

	Missile Control Effort (Integration of turn-rate)			
	Scenario#1		**Scenario#2**	
	No maneuver	9-g maneuver	No maneuver	9-g maneuver
PNG	76.2628	196.771	651.076	736.5428
VCPN	75.4792	196.0564	609.9731	698.861
Bang-bang	144.6337	240.7891	655.8412	707.0795
Bang_PN	145.7526	246.2364	639.9804	716.6012
PN-Bang	75.6336	192.4796	656.9838	692.0159

Table 6b. Missile Control Effort ($T_t = 1.0s$ and $T_m = 0.5s$) with additional drag due to missile turn-rate

Noise in LOS rate

Next, we examine the performance of the PN-Bang strategy in a noisy LOS rate environment. The results are tabulated in Table 7.

Standard Deviation	Miss Distance (m)			
	Scenario#1 (9-g maneuver)		**Scenario#2 (9-g maneuver)**	
	Drag w/o turn rate factor	Drag factoring turn-rate	Drag w/o turn rate factor	Drag factoring turn-rate
σ=0 (noise free)	0.5369	0.7763	0.3169	0.2759
σ=0.001	1.5204	1.2190	0.6493	0.2998
σ=0.005	1.3813	1.5121	0.3274	0.0432
σ=0.01	0.5901	0.6038	0.4980	0.3762
σ=0.02	7.6481	8.4838	0.2061	0.0473
σ=0.05	40.2425	41.3410	21.5640	6.3360

Table 7. PN-Bang: Miss distance with noisy LOS rate ($T_t = 1.0s$ and $T_m = 0.5s$)

We observe that the performance of PN-Bang strategy is tolerant to additive white noise in the LOS rate up to $\sigma = 0.01$ rad/s.

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VI. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

1. PROPORTIONAL NAVIGATION

PNG is known to be optimal against a non-maneuvering target, but will result in large miss-distances against evasive target maneuvers.

2. VELOCITY COMPENSATED PROPORTIONAL GUIDANCE

VCPN was found to offer little improvement in the guidance performance. The conventional PNG has a closed loop update on the closing velocity, V_c, which invariably provides update on the change in missile velocity. Hence conventional PNG in essence already factors a correction for reduced missile velocity into the guidance acceleration command.

For the case of a non-maneuvering target, PNG is known to be optimal, hence VCPN has no advantage. However, in the case of a maneuvering target, the additional velocity compensation term happens to provide a small additional acceleration at the final moment of target interception to help bring about a slight improvement in the miss distance of 0.2-0.8m. This is still not good enough to effectively intercept a maneuvering target.

3. BANG-BANG GUIDANCE

Bang-bang guidance produced much smaller miss distances but was found to be sensitive to changes in LOS rate, especially when the target is farther away. This is not desirable in the initial pursuit of the target, as it demands more control effort from the missile, which swings in either direction of the LOS due to the full 20-g swings in the guidance commands. This also makes the Bang-bang strategy vulnerable to noise in LOS rate.

4. COMBINED PROPORTIONAL NAVIGATION WITH BANG-BANG

A combined strategy incorporating proportional navigation and bang-bang guidance was investigated. It was found that the results were not good if we start with Bang-bang and then switch to PN during the terminal stage.

The better strategy, which was shown to be effective against a maneuvering target, was PN-Bang, which started the target pursuit with PN guidance and then switched to Bang-bang when the missile was nearer to the target.

This strategy worked well, since the change in LOS rate was small when the target was farther away and PN guidance was adequate, and had the advantage of less susceptibility to noise. When the missile was closer to the target, the LOS rate change was expected to be larger, especially when the target maneuvered, and Bang-bang performed well. It was also observed that the missile control effort required by the PN-Bang strategy was similar to conventional proportional navigation.

B. RECOMMENDATIONS

This strategy is thus proposed as an improvement to conventional proportional navigation guidance against an evasive target.

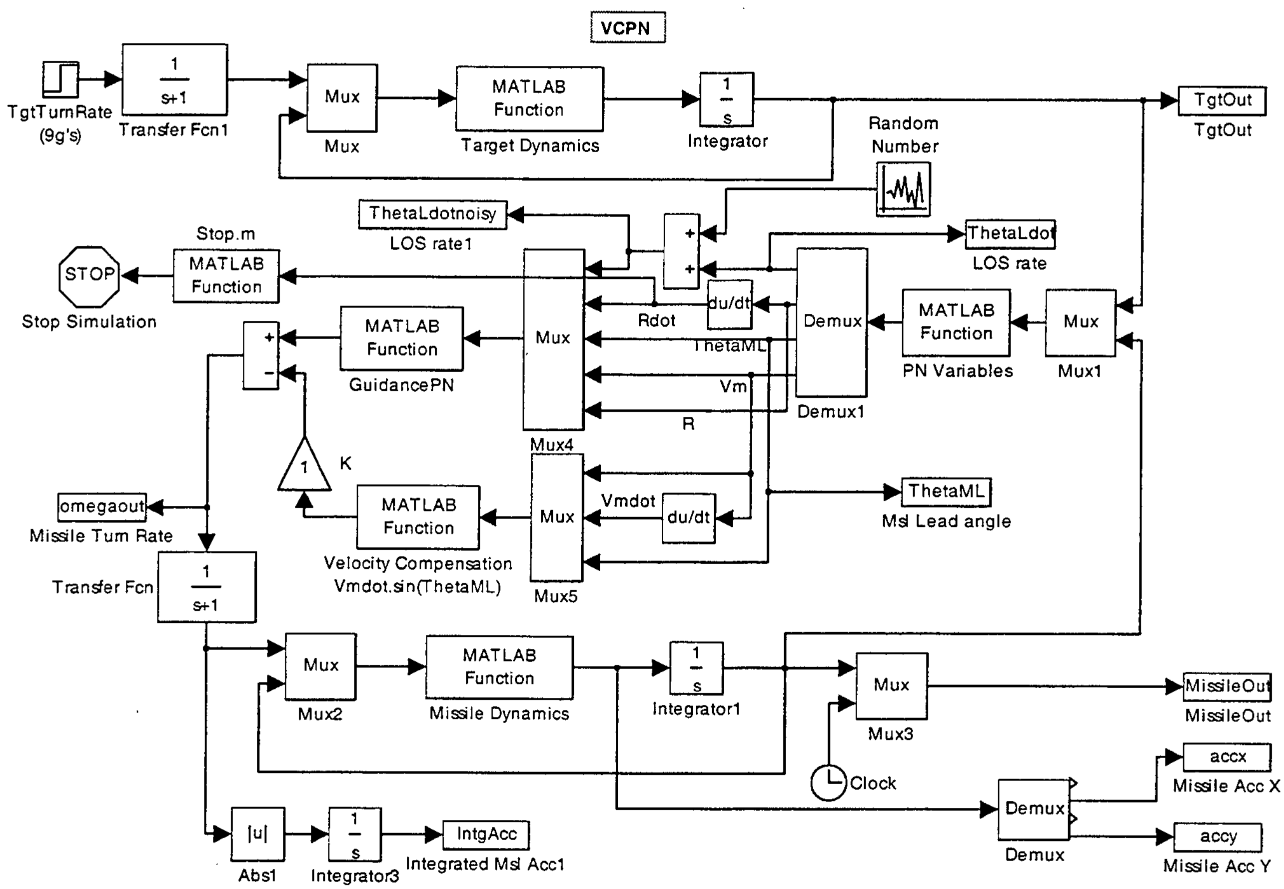
This study explored the results for two typical engagement scenarios. Follow-on studies can be carried out to test a wider engagement envelope for this guidance strategy.

APPENDIX A. SIMULINK™ MODELS

1. Proportional Navigation: (Set K=0 and Guidance Algorithm=PNG.m)

2. Velocity Compensation Proportional Navigation: (Set K=1 and Guidance Algorithm=PNG.m)

3. Bang Bang: (Set K=0 and Guidance Algorithm = Bangbang.m)

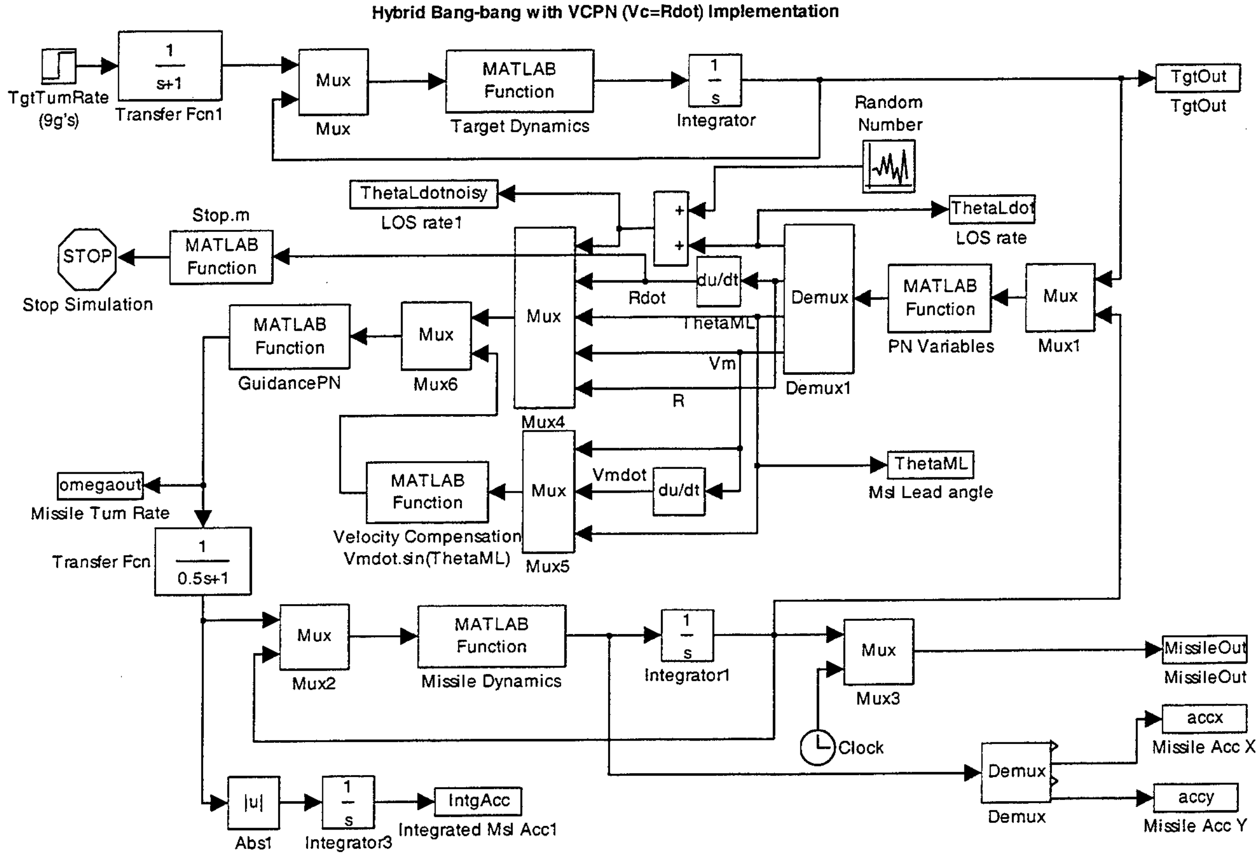


4. Hybrid Bang Bang with VCPN:

a) Starts with Bang-bang, switches to VCPN at R<2km (Bangpn.m)

b) Starts with VCPN, switches to Bang-bang at R<2km (PNBang.m)

Hybrid Bang-bang with VCPN (Vc=Rdot) Implementation



Missile Blocks:

Missile Dynamics:
MatLab Function 'MissileDynamics.m'

Missile Integrator: Initial value = minit

MissileOut: Variable Name=MissileOut
Max No. of Rows = OutputVec

Guidance Blocks:

Matlab Functions:

a) Pnvariables.m: -- Computes necessary variables
b) GuidancePN: -- Guidance Algorithm
c) Velcomp.m: -- turnrate compensation for missile velocity due to drag

Target Blocks:

TgtTurnrate Step: Step time:
i) No turn = tmax+1
ii) 9g (9x9.81/Vt) turn = intercept time – 2

Target Dynamics:
MatLab Function 'TwoDdynamics.m'

Target Integrator: Initial value = tinit

TgtOut: Variable Name=MissileOut
Max No. of Rows = OutputVec

APPENDIX B. SOURCE CODES

The list of Matlab source codes are:

S/N	Filename	Task
1.	Init.m	Initializes the following: a) missile and target state vectors b) Missile Drag variables c) Missile's initial launch velocity d) Sampling time and maximum missile flight time
2.	Stop.m	Stops the simulation run when missile passes target. This occurs when closing velocity is +ve (set at Vc>=100m/s).
3.	Missiledynamics.m	Computes missile state vector given the commanded turn rate. It considers the deceleration in the missile's coasting speed due to aerodynamic drag. The drag also takes into account the effect of turn rate.
4.	Targetdynamics.m	Computes the target state vector given the commanded turn rate. No drag component is considered as it is assumed that the aircraft has sustained thrust to overcome its drag components.
5.	PNvariables.m	Computes the variables for Proportional Navigation missile guidance. Inputs: Missile and Target State Vectors. (X-pos, X-vel, Y-pos, Y-vel) Returns: [ThetaLdot,R,ThetaML,Vm]
6.	Guidance Algorithm: a) PNG.m b) Bangbang.m c) BangPN.m d) PNbang.m	Computes the guidance command (msl turn rate) subject to 20-g limit. a) Conventional True Prop. Nav. b) Bang-bang guidance. c) Starts with 20-g Bang-bang and switches to Prop Nav with N'=3 when target is within 2km. d) Starts with Prop Nav with N'=3; switches to 20-g Bang-Bang when target is within 2km.
7.	Velcomp.m	Computes the additional missile turn rate to compensate for reduced speed of missile due to Drag.
8.	Plotfigures.m	Plots: Fig. 1: Flight trajectories of missile and target Fig. 2: Miss distance Fig. 3: Missile Speed profile Fig. 4: Missile acceleration profile Fig. 5: Missile acceleration control effort (Integration of acceleration commands)

1. Initialization

```
%% Init.m
%  This script file initializes the point mass missile intercept simulation

global Vcinit beta

tmax=11;                       % Seconds - max. total simulation time
stime = 0.001;                 % Sampling time
OutputVec=[20000,1,stime]; % For output vector


% Computing for Missile Drag
h=2000;            % height of missile during cruise
radius=0.15;       % radius of missile body
Area=pi*radius^2; % Cross-sectional Area of missile
CD=0.2;            % Coefficient of Drag
Mass=300;

if h<9144       % below 9144m
   rho=1.22557*exp(-h/9144);
else
   rho=1.75228763*exp(-h/6705.6);
end

beta=rho*CD*Area/(2*Mass);     %Drag=beta*Vm^2;
                               %Vmdot=Drag/Mass (deceleration of Vm)

% Initialize Target State Vector
% t1= scenario#1 and t2=scenario#2

Vt=500;

t1=[  10000;             % x-position
         -Vt;            % x-velocity
         1000;           % y-position
         0];             % y-velocity

t2=[  8000;              % x-position
      -Vt*sin(pi/4);     % x-velocity
      800;               % y-position
      Vt*sin(pi/4)];     % y-velocity

scenario= input('Enter scenario no. "1" or "2" : ');

if scenario == 1
   tinit=t1;
else
   if scenario==2
      tinit=t2;
   else
   scenrario= input('Please re-enter scenario no. "1" or "2" : ');
   end
end
```

```
% Initialize Missile State Vector
% Missile is initially twice the speed of target

Vm=2*Vt;

minit=[  0;                                        % x-position
         Vm*cos(atan2(tinit(3),tinit(1)));         % x-velocity
         0;                                        % y-position
         Vm*sin(atan2(tinit(3),tinit(1)))];        % y-velocity

% Calculate initial closing velocity for Approx#2

LOSinit=atan2(tinit(3),tinit(1));          % Initial LOS Target to Missile
Vminit=sqrt(minit(4)^2+minit(2)^2);        % Initial Missile velocity
Vtinit=sqrt(tinit(4)^2+tinit(2)^2);        % Initial Target velocity
ThetaMinit=atan2(minit(4),minit(2));       % Initial Missile heading
ThetaTinit=atan2(tinit(4),tinit(2));       % Initial Target heading

Vcinit=Vtinit*cos(ThetaTinit-LOSinit)-Vminit*cos(ThetaMinit-LOSinit)
```

2. Stopping Simulation

```
% Stop.m
% Stop Simulation if Vc=+ve ... ie missile pass target.

function [stop] = stop(u)

% Checks if Vc is closing or not
if u(1)>=100
   stop=1;
else
   stop=0;
end
```

3. **Missile Dynamics**

```
% Missiledynamics.m : Matlab Function used in Simulink model "PNBang.mdl"
% Computes the 2-Dimension Missile dynamics.
% Inputs: State Vector (X-position, X-velocity, Y-position, Y-velocity)
% Returns: Missile Turn Rate

function [xdot] = Missiledynamics(u)

global beta

w=u(1);            % turnrate
x=u(2:5);          % state vector : x(t)

Vm=sqrt((x(2)^2+x(4)^2)); % Missile velcoity, Vm;

A=[0, 1, 0, 0;
   0, 0, 0,-w;
   0, 0, 0, 1;
   0, w, 0, 0];

% Compute Drag
% Nominal frontal Drag due to aerodynamic surface
Drag0=[0; x(2)*beta*Vm; 0; x(4)*beta*Vm];

% Assuming Drag_max is about 5xDrag0 at maximum turnrate (i.e. w_max)

w_max=20*9.81/Vm;       % Max. turn rate = 20g's
k=4/(w_max^2);
Drag=Drag0*(1+k*w^2);   % Drag is a function of turnrate.

% Output state vector : xdot=A(w).x - Drag

xdot=A*x-Drag;
```

4. **Target Dynamics**

```
% Targetdynamics.m : Matlab Function used in Simulink model "PNBang.mdl"
% Computes the 2-Dimension Target dynamics.
% Inputs: State Vector (X-position, X-velocity, Y-position, Y-velocity)
% Returns: Target Turn Rate

function [xdot] = PNdynamics(u)

w=u(1);            % turnrate
x=u(2:5);    % state vector : x(t)

% Output state vector : xdot=A(w).x

A=[0, 1, 0, 0;
   0, 0, 0,-w;
   0, 0, 0, 1;
   0, w, 0, 0];

xdot=A*x;
```

5. Proportional Navigation Variables

```
% PNvariables.m
% Matlab Function used in Simulink model "PNBang.mdl"
% Computes the variables for Proportional Navigation missile guidance.
% Inputs: Missile and Target State Vectors.(X-pos, X-vel, Y-pos, Y-vel)
% Returns: [ThetaLdot,R,ThetaML,Vm]

function [output] = PNvariables(u)

% Compute the LOS (missile-target) turnrate
TgtOut=u(1:4);
MissileOut=u(5:8);

xt=TgtOut(1);  % Target x-position
Vxt=TgtOut(2); % Target x-direction velocity
yt=TgtOut(3);  % Target y-position
Vyt=TgtOut(4); % Target y-direction velocity

xm=MissileOut(1); % Missile x-position
Vxm=MissileOut(2);% Missile x-direction velocity
ym=MissileOut(3); % Missile y-position
Vym=MissileOut(4);% Missile y-direction velocity

% Compute ThetaL: LOS angle between missile and target
ThetaL=atan2((yt-ym),(xt-xm));

% Compute ThetaM: Missile heading
ThetaM=atan2(Vym,Vxm);

% Difference of angle between Missile Heading to LOS (Msl-Tgt)
ThetaML = ThetaM-ThetaL;

% Compute R: Range of target to missile
Rsquare=(xt-xm)^2+(yt-ym)^2;
R=sqrt(Rsquare);

% Missile velocity
Vm=sqrt(Vxm^2+Vym^2);

% Compute Thetadot
ThetaLdot=((xt-xm)*(Vyt-Vym)-(yt-ym)*(Vxt-Vxm))/Rsquare;

% PN variables output
output=[ThetaLdot;R;ThetaML;Vm];
```

6. Guidance Algorithm

a) PNG.m: Conventional True Proportional Navigation

```
% PNG.m
% Matlab Function used in Simulink file "PNguidance"
% Computes the guidance command (missile turn rate) for True Prop Nav
% Inputs:
% u(1)= ThetaLdot (rate of change of LOS)
% u(2)= Rdot=Vc (closing velocity)
% u(3)= ThetaML (missile leading angle)
% u(4)= Vm (missile velocity)
% u(5)= R  (Range of target from msl)
% Returns: Missile Turn Rate based on PN

function [omegaout] = PNG(u)

% Compute the missile turnrate
omegaout1=-u(2)*u(1);    % Vc*ThetaLdot/Vm*cos(ThetaML)

% PropNav: N'*Vc*ThetaLdot/Vm*cos(ThetaML)
omega=3*omegaout1/(u(4)*cos(u(3)));

% 20-g limit
g_limit=20*9.81*sign(omegaout1)/u(4);
if abs(omega) >= abs(g_limit)
   omegaout=g_limit;
else
   omegaout=omega;
end
```

b) Bangbang.m: 20-g Bang bang guidance

```
% Bangbang.m
% Matlab Function used in Simulink file "BangBang"
% Computes the guidance command (missile turn rate) for PN Bang-bang
missile guidance.
% Inputs:
% u(1)= ThetaLdot (rate of change of LOS)
% u(2)= Rdot=Vc (closing velocity)
% u(3)= ThetaML (missile leading angle)
% u(4)= Vm (missile velocity)
% Returns: Missile Turn Rate based on PN

function [omegaout] = Bangbang(u)

% Compute the missile turnrate
omegaout1=-u(2)*u(1);    % Vc*ThetaLdot/Vm*cos(ThetaML)
omegaout=20*9.81*sign(omegaout1)/u(4);
```

c) BangPN.m: Starts with Bang-bang; switches to Prop Nav at Range<2km

```
% Bangpn.m
% Matlab Function used in Simulink file "Hybrid BangPN"
% Inputs:
% u(1)= ThetaLdot (rate of change of LOS)
% u(2)= Rdot=Vc (closing velocity)
% u(3)= ThetaML (missile leading angle)
% u(4)= Vm (missile velocity)
% u(5)= R (Range of target from missile)
% u(6)= Vmdotsin(ThetaML)/cos(ThetaML) = Velocity compensation
% Returns: Missile Turn Rate based on PN

function [omegaout] = Bangpn(u)

% Compute the missile turnrate
% For TPN Implementation
omegaout1=-u(2)*u(1);    % Vc*ThetaLdot/Vm*cos(ThetaML)

if u(5)<2000
   omega=3*omegaout1/(u(4)*cos(u(3)))-u(6); % PropNav:
N'*Vc*ThetaLdot/Vm*cos(ThetaML)
   % 20-g limit
   g_limit=20*9.81*sign(omegaout1)/u(4);
   if abs(omega) >= abs(g_limit)
   omegaout=g_limit;
   else
   omegaout=omega;
end

else
   omegaout=20*9.81*sign(omegaout1)/u(4); % bang bang
```

d) PNBang.m: Starts with Prop Nav and switches to Bang-bang at Range <2km

```
% PNBang.m
% Matlab Function used in Simulink file "Hybrid PNBang"
% Inputs:
% u(1)= ThetaLdot (rate of change of LOS)
% u(2)= Rdot=Vc (closing velocity)
% u(3)= ThetaML (missile leading angle)
% u(4)= Vm (missile velocity)
% u(5)= R (Range of target from missile)
% u(6)= Vmdotsin(ThetaML)/cos(ThetaML) = Velocity compensation
% Returns: Missile Turn Rate based on PN

function [omegaout] = PNbang(u)

% Compute the missile turnrate
% For TPN Implementation
omegaout1=-u(2)*u(1);    % Vc*ThetaLdot/Vm*cos(ThetaML)

if u(5)>2000
   omega=3*omegaout1/(u(4)*cos(u(3)))-u(6); % PropNav:
N'*Vc*ThetaLdot/Vm*cos(ThetaML)
```

```
    % 20-g limit
    g_limit=20*9.81*sign(omegaout1)/u(4);
    if abs(omega) >= abs(g_limit)
    omegaout=g_limit;
    else
    omegaout=omega;
end

else
    omegaout=20*9.81*sign(omegaout1)/u(4); % bang bang
```

7. Velocity Compensation

```
% Velcomp.m
% Matlab Function used in PN Bang-Bang guidance
% Computes the additional missile turn rate to compensate for reduced Vm
% Inputs:   u(1)= Vm (missile velocity)
%           u(2)= Vmdot (missile deceleration)
%           u(3)= ThetaML (missile leading angle)
% Returns:  Missile Turn Rate Compensation
%           [Vmdot*sin(ThetaML)/Vm*cos(ThetaML)]

function [comp] = Velcomp(u)

% Compute the missile turnrate compensation due to reduction in Vm

comp=u(2)*tan(u(3))/u(1);      % Vmdot*sin(ThetaML)/Vm*cos(ThetaML)
```

8. Plotting

```
%%  Plotfigures.m
%  To plot:
%  Fig. 1: Flight trajectories of missile and target
%  Fig. 2: Miss distance
%  Fig. 3: Missile Speed profile
%  Fig. 4: Missile acceleration profile
%  Fig. 5: Missile acceleration control effort (Integration of acceleration
commands)

Missilepos=[MissileOut(:,1),MissileOut(:,3)];
Tgtpos=[TgtOut(:,1),TgtOut(:,3)];
time=MissileOut(:,5);
Mslaccx=accx/9.81;
Mslaccy=accy/9.81;

% Compute:
% a) Miss Distance
% b) Vm
% c) Missile Accelerations

missdis=[];
Vm=[];
```

```
Mslacc=[];

for i=1:max(size(MissileOut)),

   % a) Miss Distance
   miss=norm(Missilepos(i,:)-Tgtpos(i,:));
   missdis=[missdis,miss];

   % b) Vm
   Vm1=(MissileOut(i,2)^2+MissileOut(i,4)^2)^0.5;
   Vm=[Vm,Vm1];


   % c) Missile Accelerations
   Am=sqrt(Mslaccx(i)^2+Mslaccy(i)^2);
   ThetaM=atan2(MissileOut(i,4),MissileOut(i,2));
   Theta=atan2(Mslaccy(i),Mslaccx(i));
   acc=Am*sin(Theta-ThetaM);
   Mslacc=[Mslacc,acc];

end

mindis=min(missdis)  % Minimum Miss Distance (i.e. Closest Pt of Approach)
index=find(missdis==mindis);  % Index where min. distanc occurs
intercept=sqrt(Tgtpos(index,1)^2+Tgtpos(index,2)^2); % Intercept point
Controleffort=IntgAcc(1:index).*Vm(1:index)';    % Acc Control Effort


figure(1)
clf
plot(Tgtpos(:,1),Tgtpos(:,2),'r')
hold on
plot(Missilepos(:,1),Missilepos(:,2))
plot(Tgtpos(index,1),Tgtpos(index,2),'x')
plot(Missilepos(index,1),Missilepos(index,2),'o')
title('Rear Cross-Range Scenario Geometry')
xlabel('x-coordinates (meters)')
ylabel('y-coordinates (meters)')
gtext('Missile')
gtext('Target')
gtext(['Intercept range =',num2str(intercept)])

figure(2)
clf
plot(time,missdis)
title('Rear cross-Range Scenario: Miss Distance')
xlabel('time (sec)')
ylabel('Miss Disttance (meters)')
gtext(['Miss-distance(m)=',num2str(mindis),'    Time of Intercept
(s)=',num2str((index-1)*stime)])

figure(3)
clf
plot(time,Vm)
title('Rear cross-Range Scenario: Missile Velocity')
xlabel('time (seconds)')
ylabel('Missile vector velocity, Vm (meters/sec)')
```

```
grid

figure(4)
clf
plot(time(1:index),Mslacc(1:index))
title('Missile Absolute Acceleration')
xlabel('time (sec)')
ylabel('Acceleration in g')
%gtext(['Missile Initial Accelertion =',num2str(Mslacc(1))])
gtext(['Missile Acceleration Effort=',num2str(Controleffort(index))])
grid


figure(5)
clf
plot(time(1:index),Mslaccx(1:index),'-',time(1:index),Mslaccy(1:index),'-')
legend('accx','accy')
grid
title('Missile Acceleration in x-y directions')
xlabel('time (sec)')
ylabel('Acceleration in g')

figure(6)
clf
plot(time(1:index),Controleffort,'-')
title('Integrated Missile Control Effort')
xlabel('time (sec)')
ylabel('Acceleration in g')
gtext(['Missile Acceleration Effort=',num2str(Controleffort(index))])
hold on
```

APPENDIX C. SIMULATION RESULTS

This appendix contains the following plots for the different guidance strategies:

Fig. 1: Flight trajectories of missile and target

Fig. 2: Miss distance

Fig. 3: Missile speed profile

Fig. 4: Missile acceleration profile and control effort

The guidance strategies are:

1. PN with different turn-rate time constants
2. VCPN
3. Bang-bang
4. Bang-bang with PN: Starts with Bang-bang switching to PN when R<2km
5. PN with Bang-bang: Starts with PN switching to Bang-bang when R<2km

1. PN with different turn-rate time constants

Scenario #1

Fig. 1 Flight Trajectory

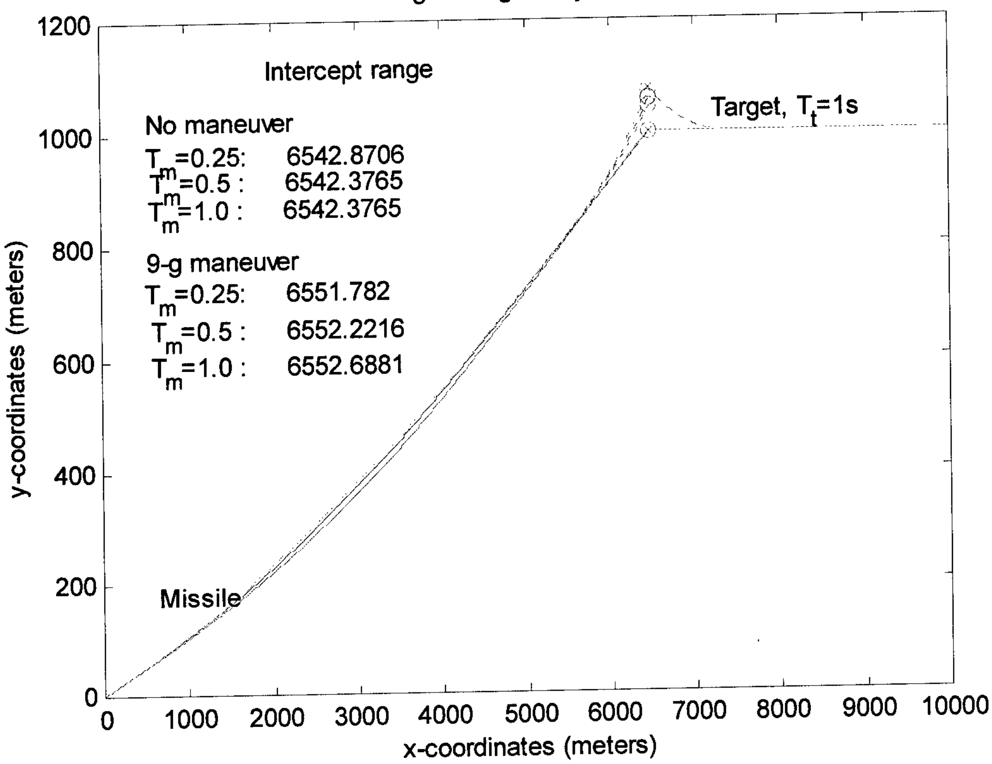


Fig. 2 Miss Distance

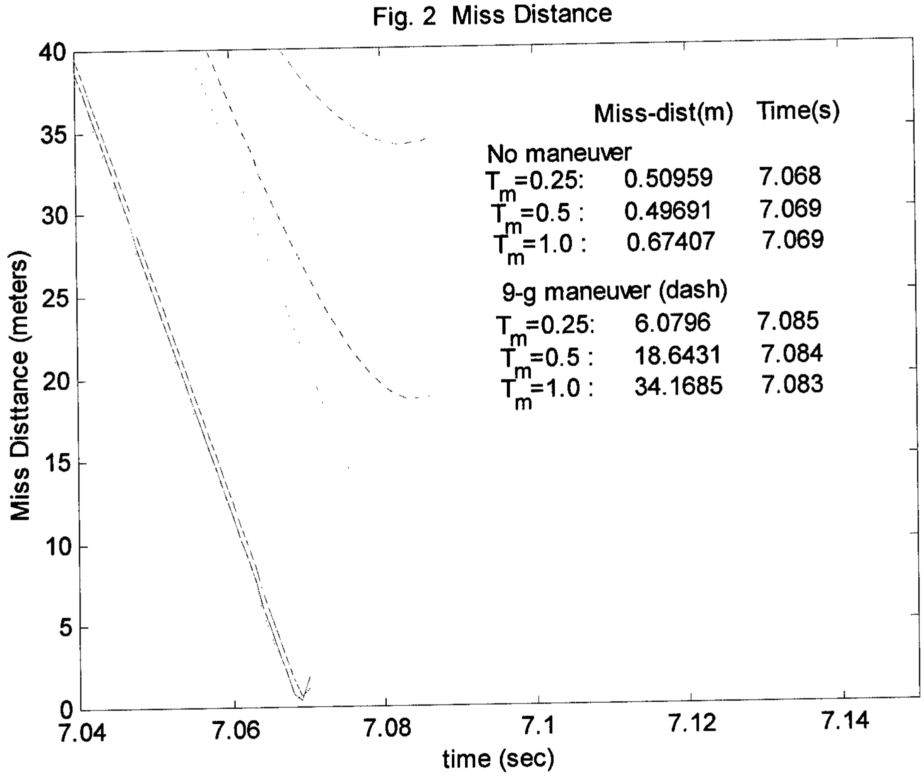
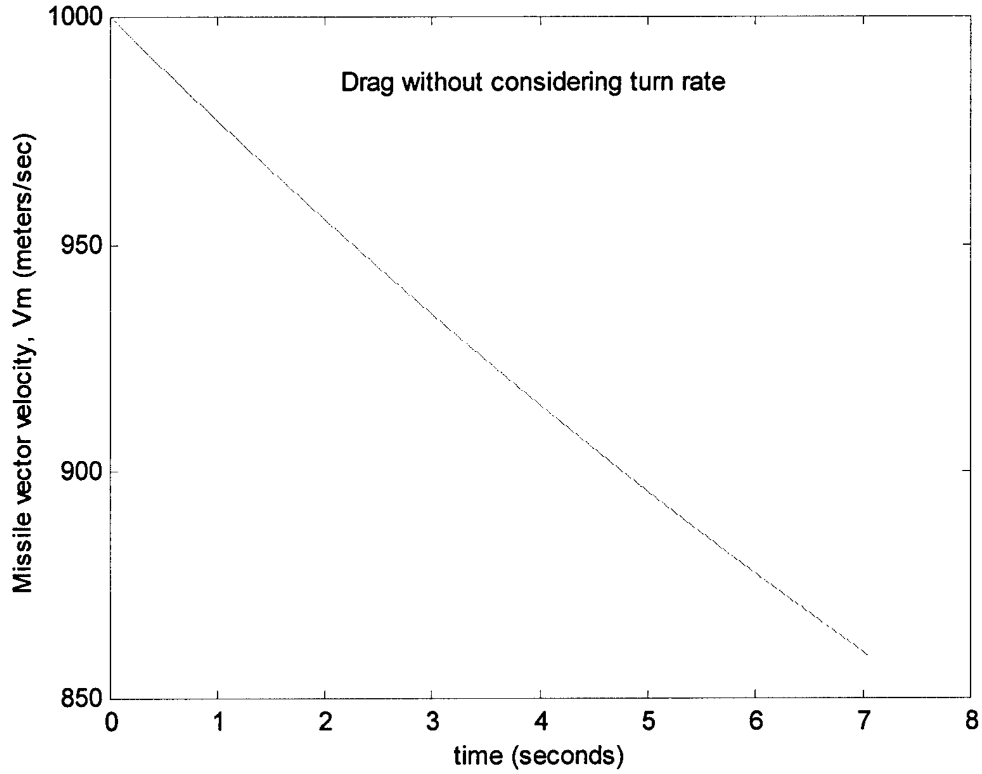
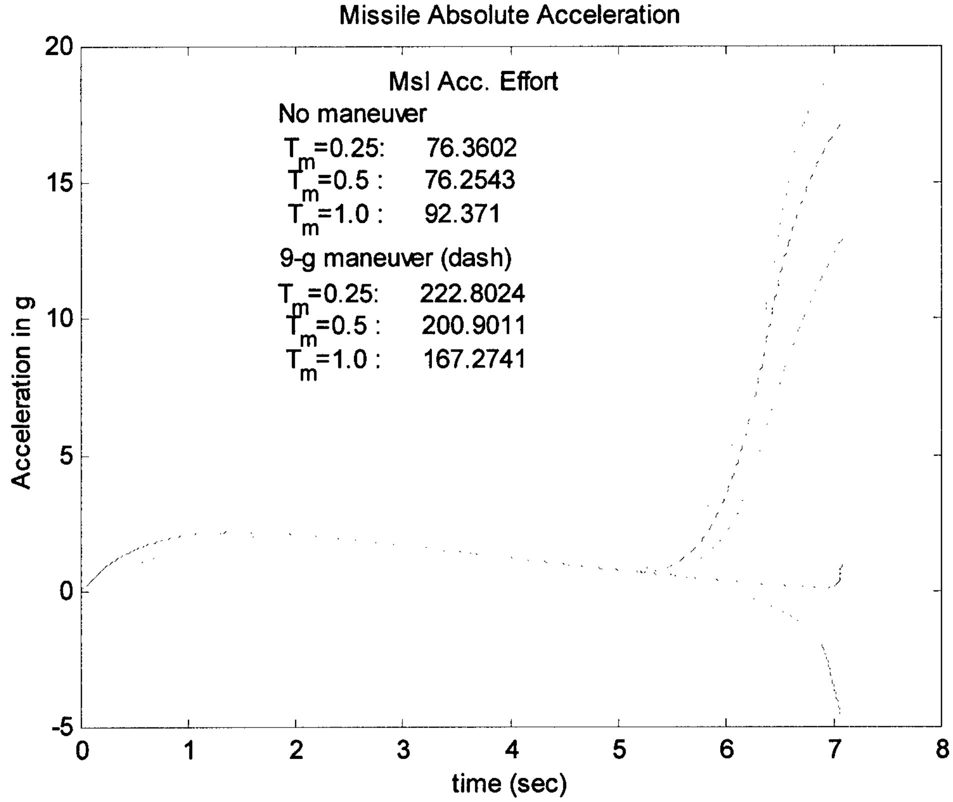


Fig. 3 Missile Velocity



Missile Absolute Acceleration



Scenario #2

Fig. 1 Flight Trajectory

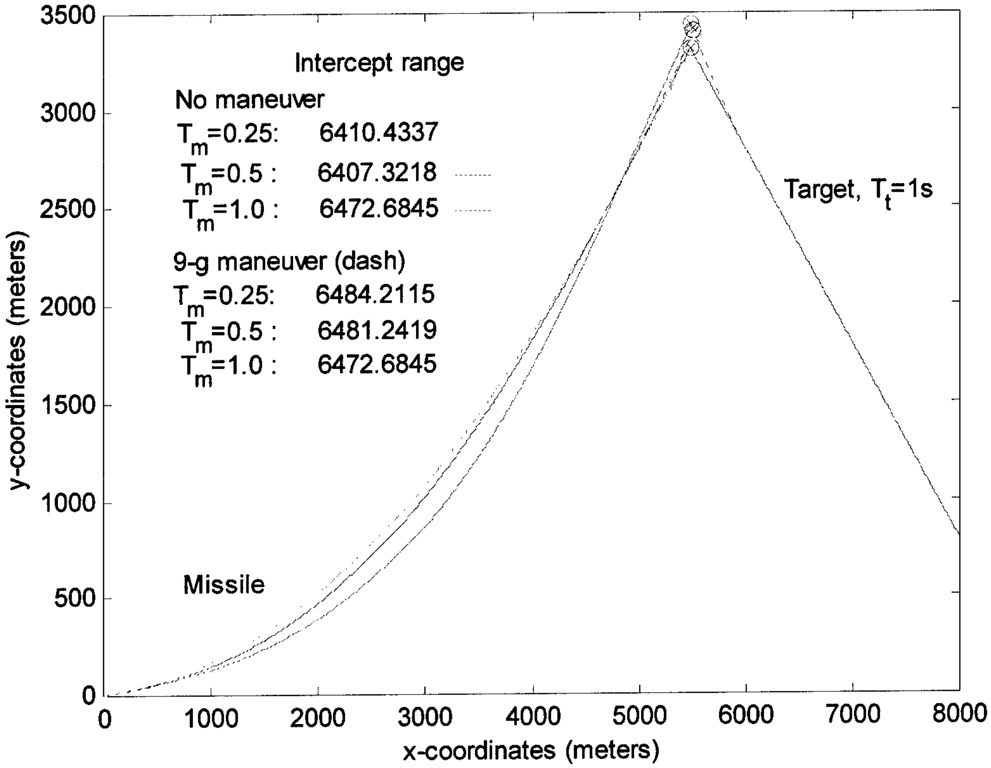


Fig. 2 Miss Distance

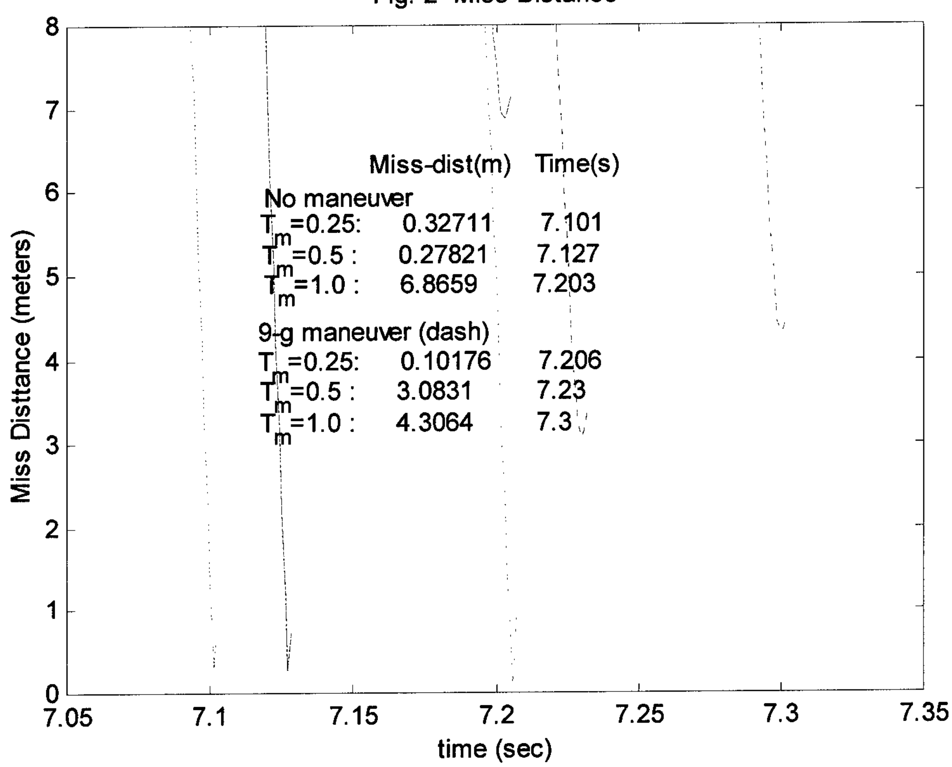
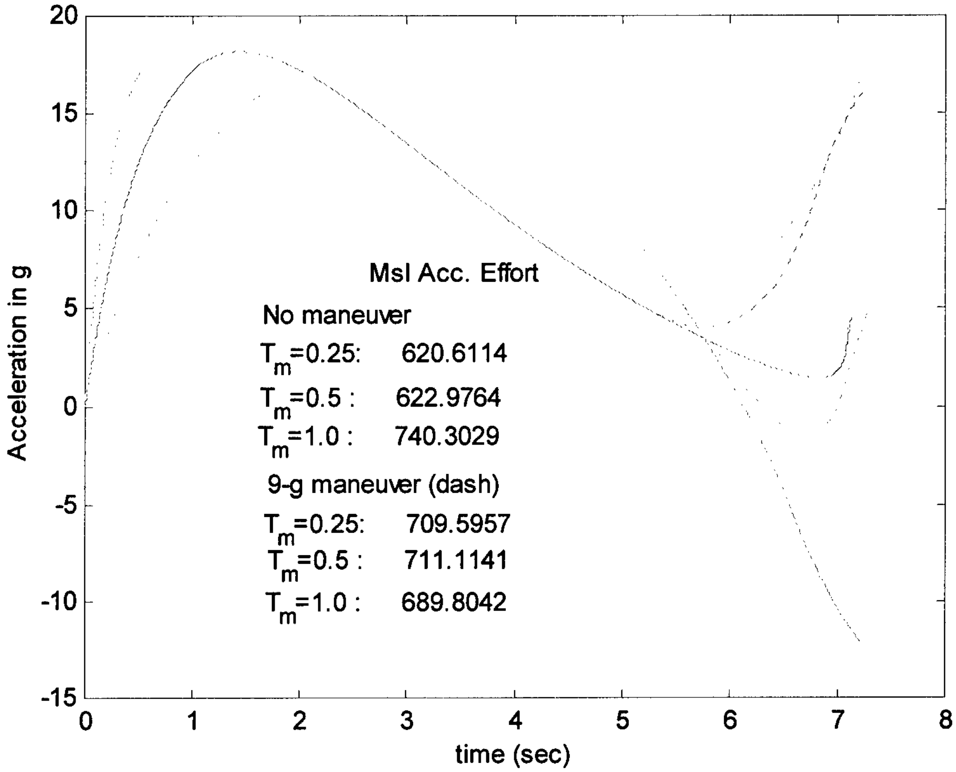


Fig. 4 Missile Acceleration Profile



2. VCPN

Scenario #1 (with nominal drag)

Fig. 1 Flight Trajectory

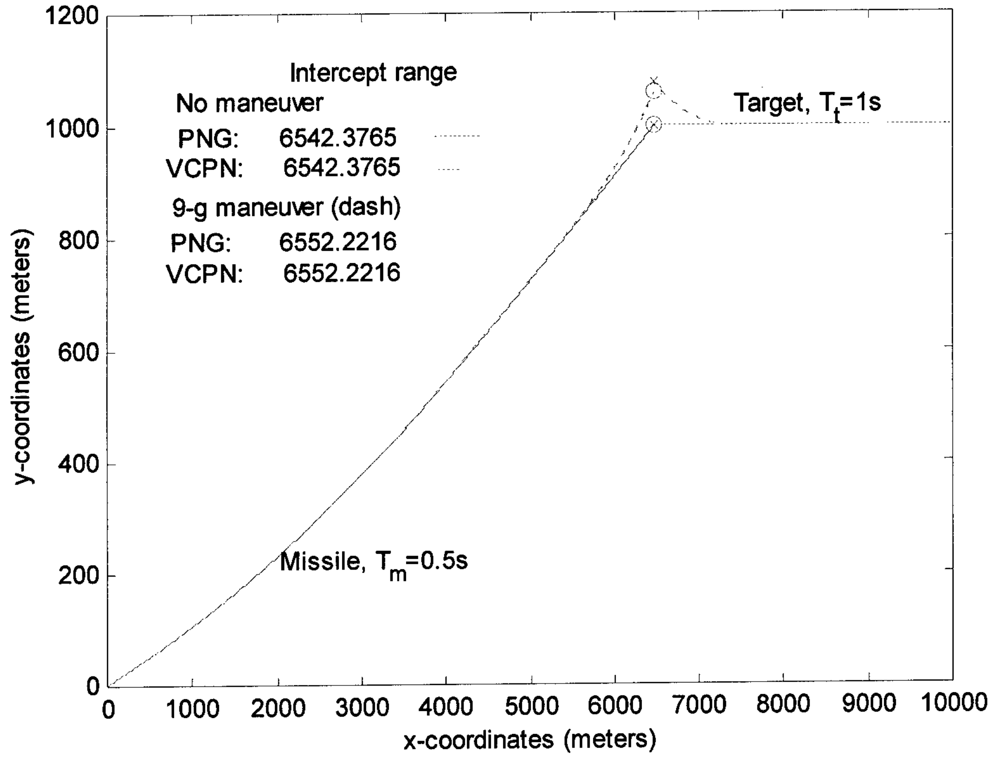
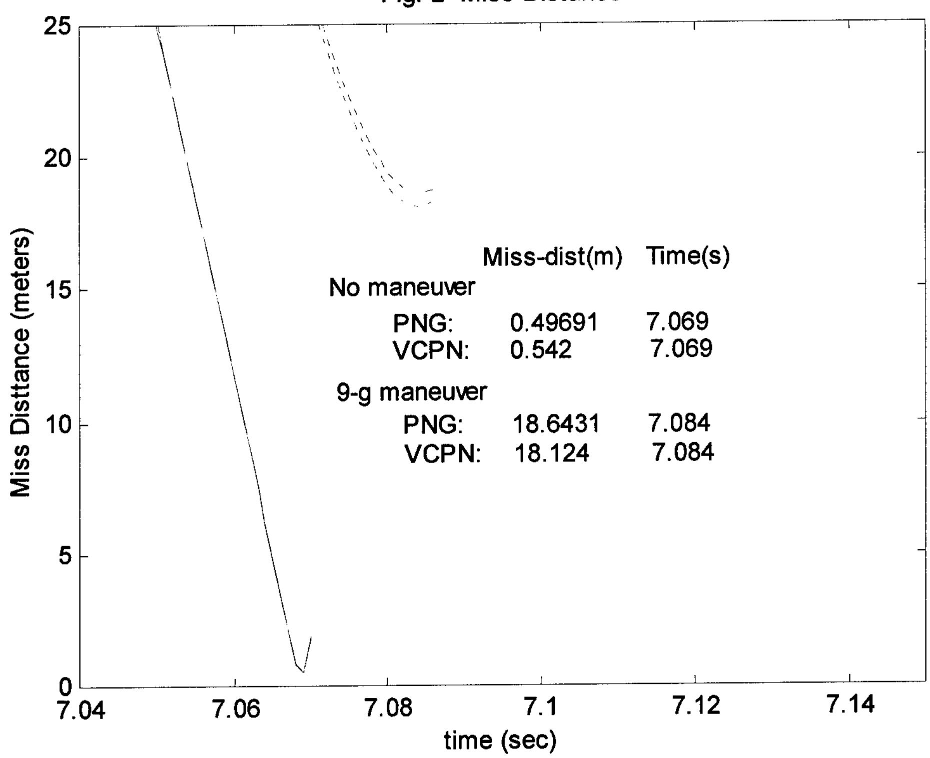


Fig. 2 Miss Distance



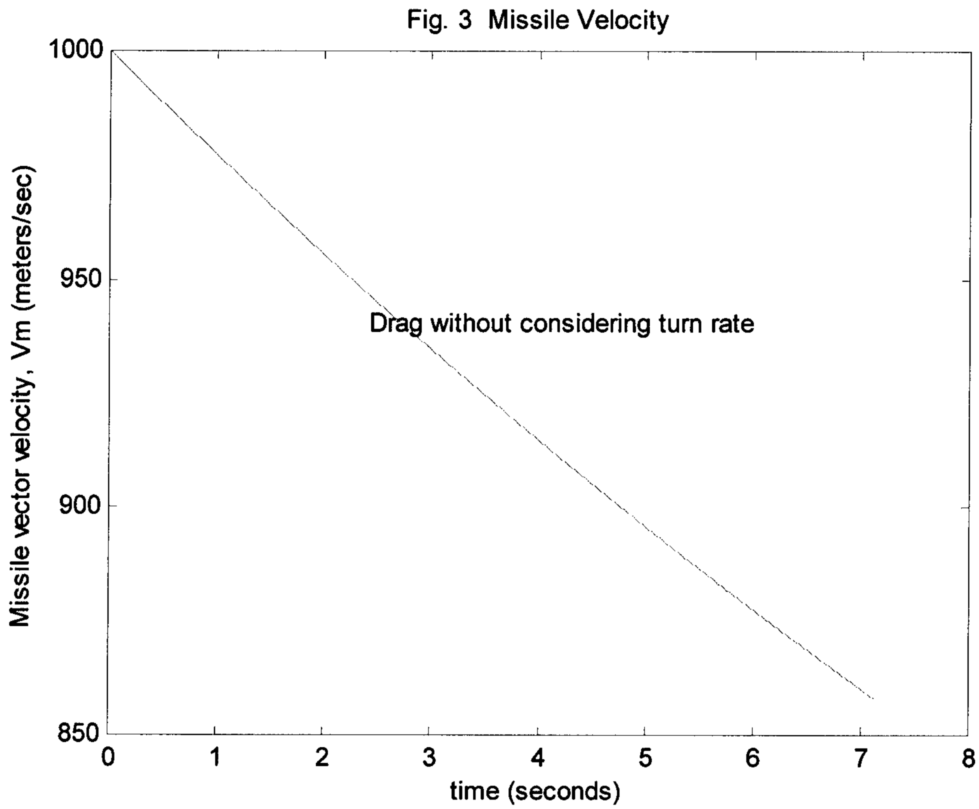


Fig. 3 Missile Velocity
1000
950
900
850
0
1
2
3
4
5
6
7
8
Missile vector velocity, Vm (meters/sec)
time (seconds)
Drag without considering turn rate

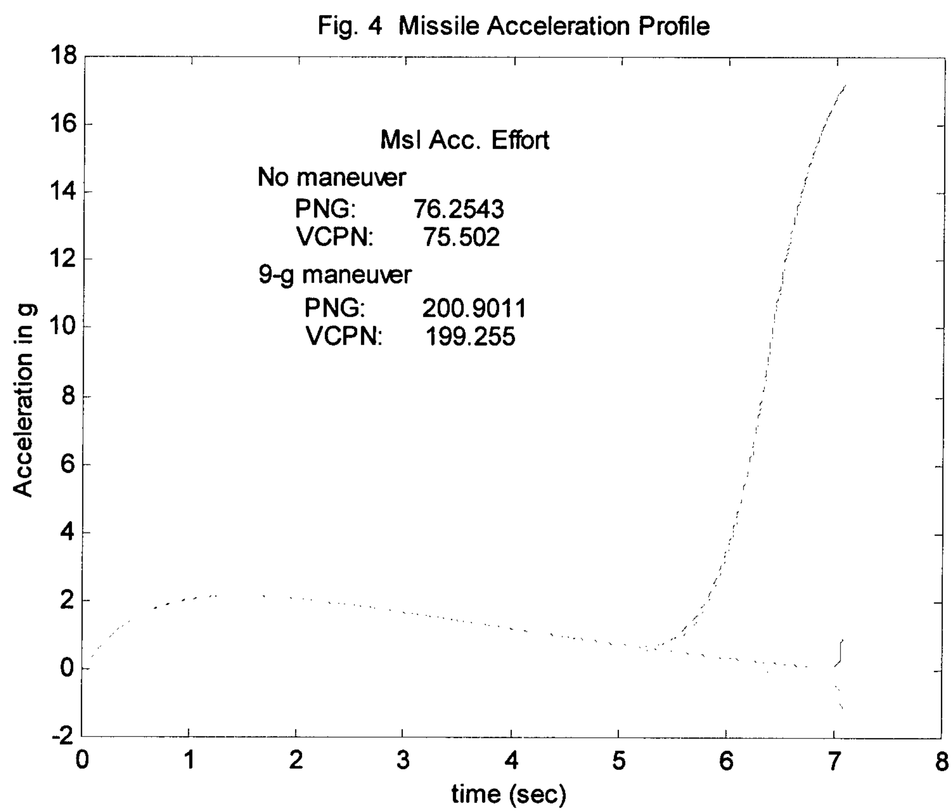


Fig. 4 Missile Acceleration Profile
18
16
14
12
10
8
6
4
2
0
-2
0
1
2
3
4
5
6
7
8
Acceleration in g
time (sec)
Msl Acc. Effort
No maneuver
PNG: 76.2543
VCPN: 75.502
9-g maneuver
PNG: 200.9011
VCPN: 199.255

Scenario #2 (with nominal drag)

Fig. 1 Flight Trajectory

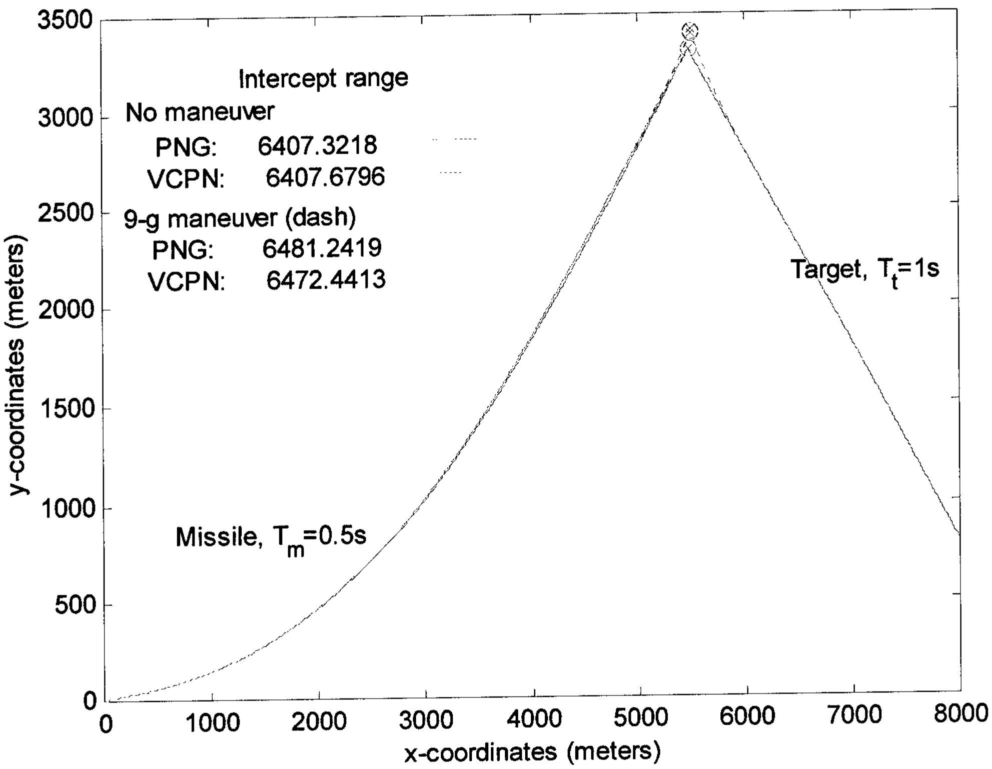
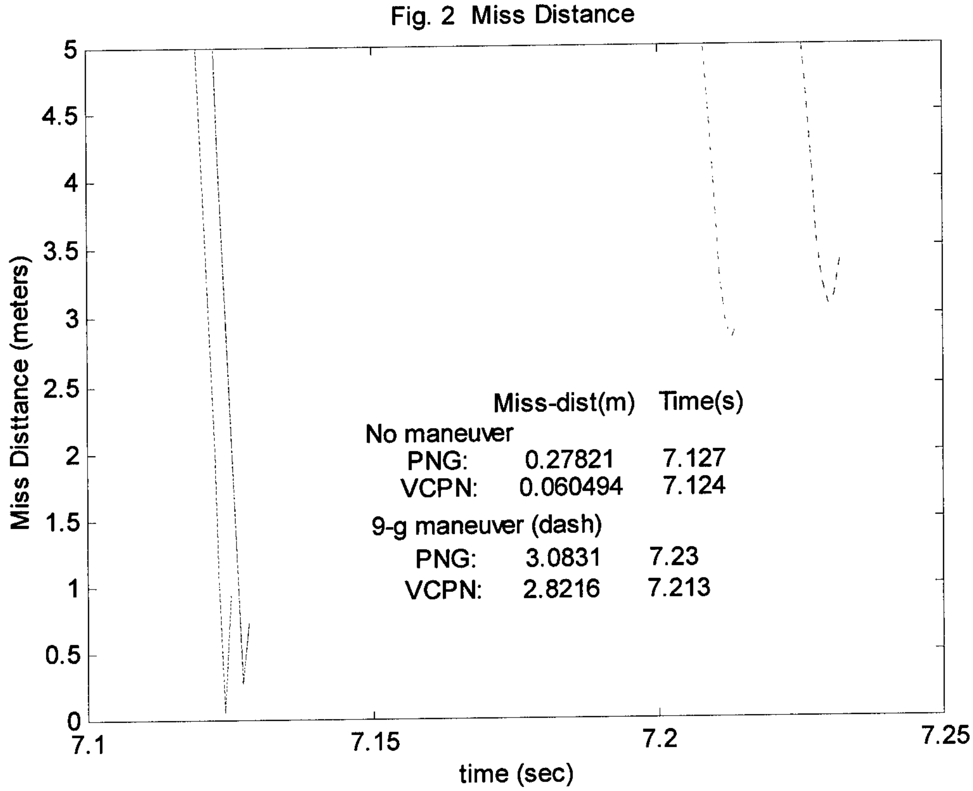


Fig. 2 Miss Distance



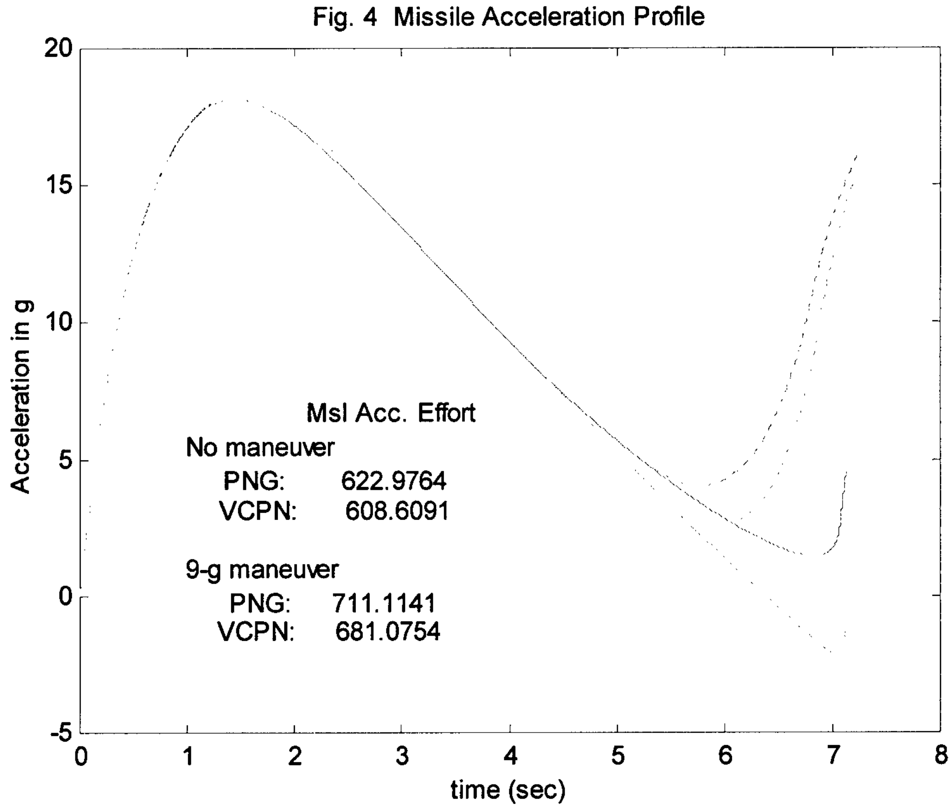


Fig. 4 Missile Acceleration Profile
Acceleration in g
time (sec)
20
15
10
5
0
-5
0
1
2
3
4
5
6
7
8
Msl Acc. Effort
No maneuver
PNG: 622.9764
VCPN: 608.6091
9-g maneuver
PNG: 711.1141
VCPN: 681.0754

Scenario #1 (with additional drag due to turn-rate)

Fig. 1 Flight Trajectory

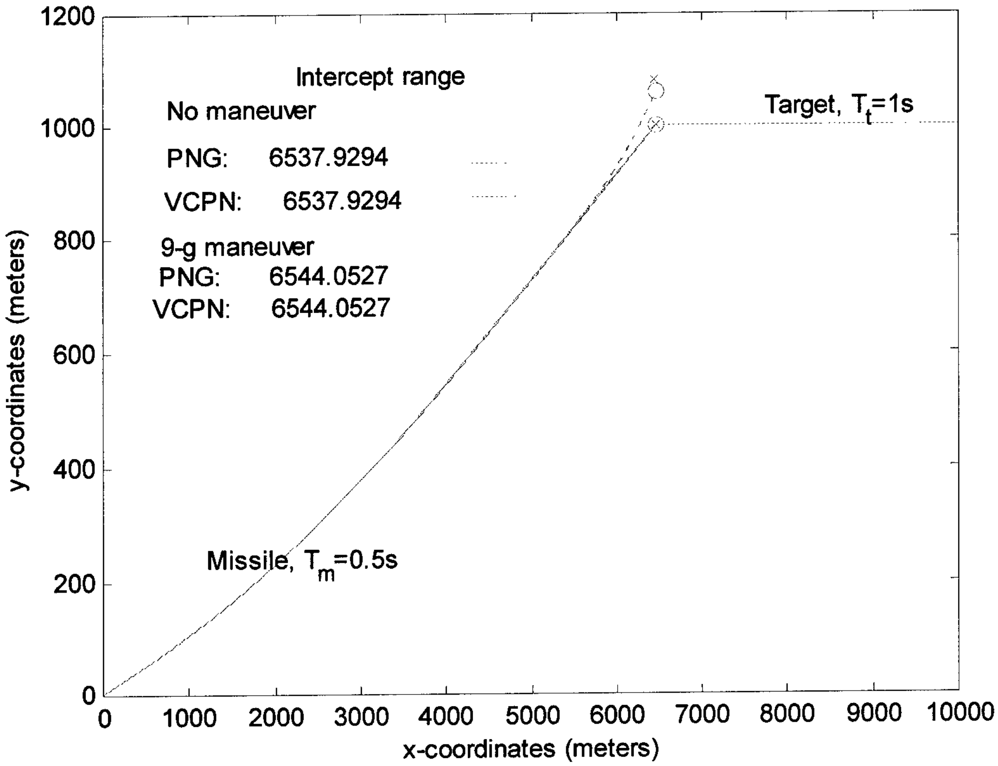


Fig. 2 Miss Distance

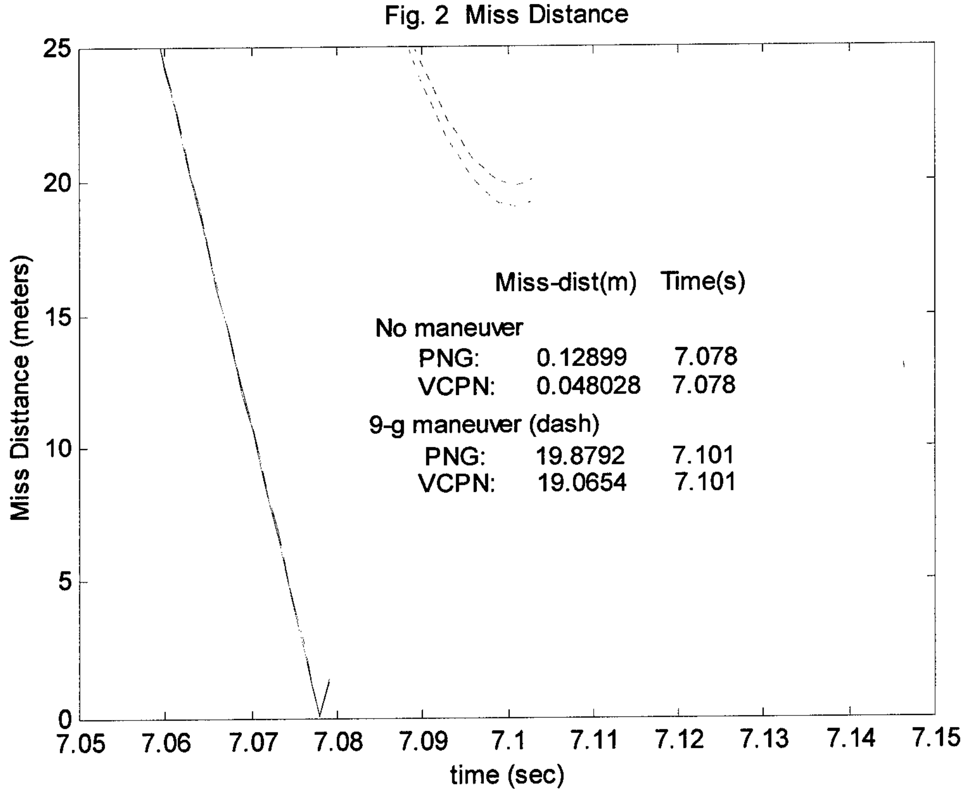


Fig. 3 Missile Velocity

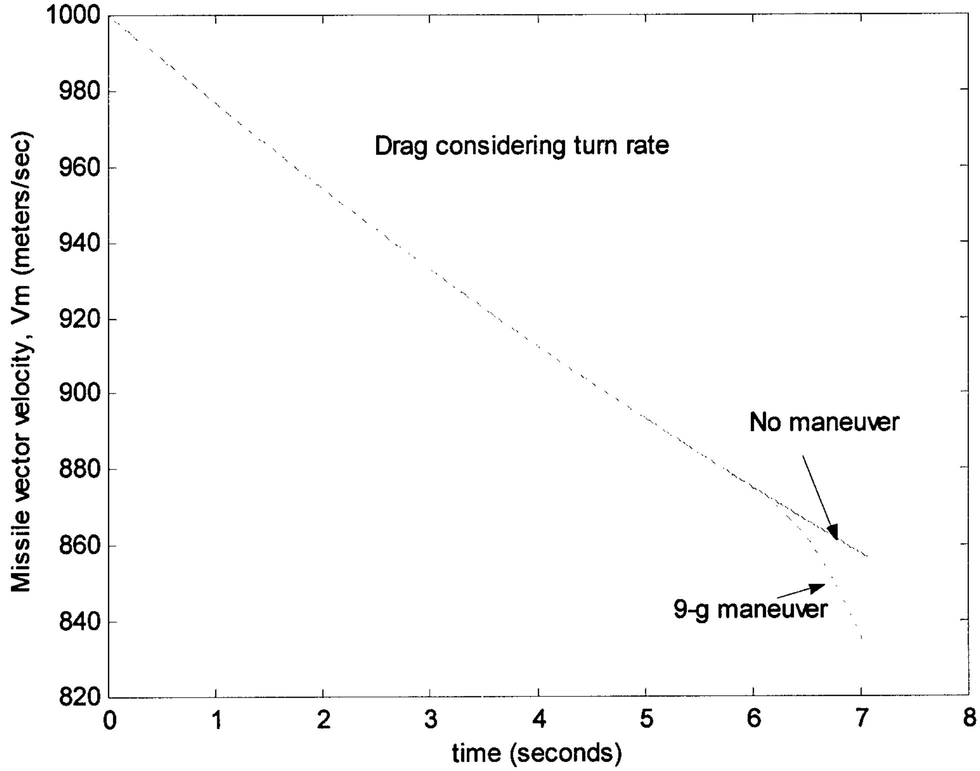
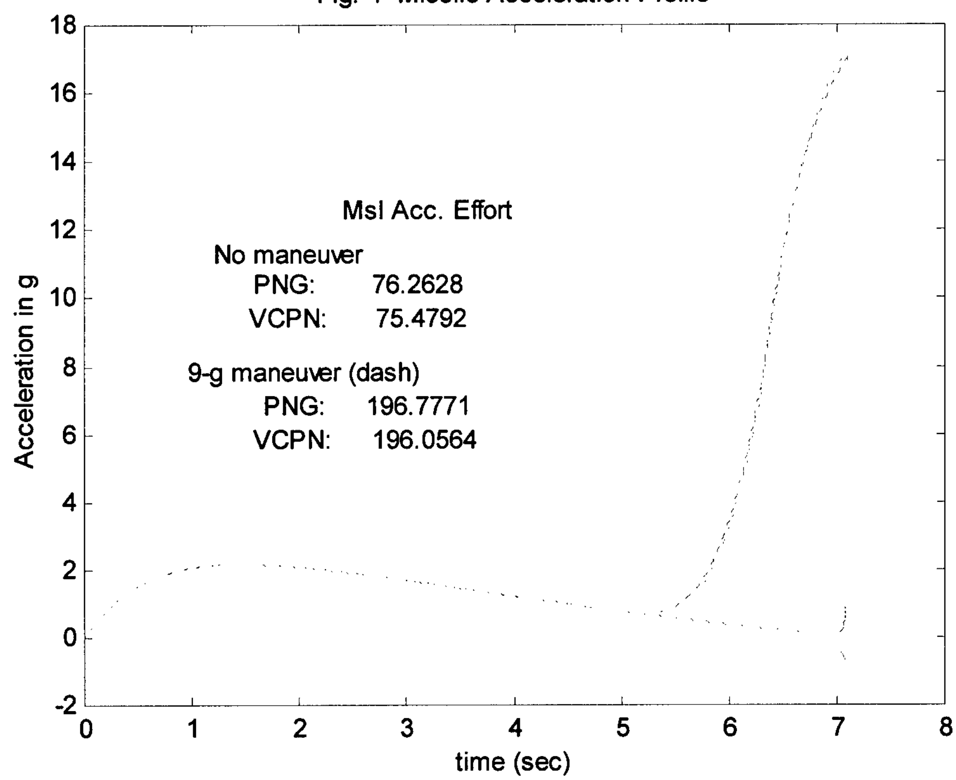
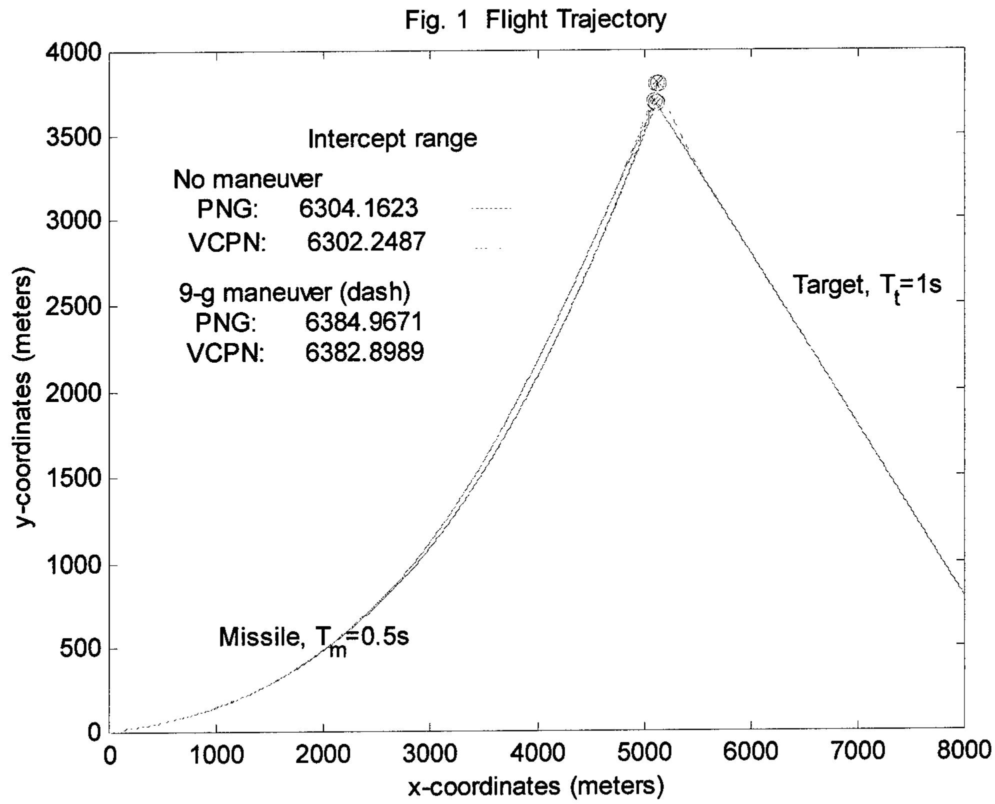


Fig. 4 Missile Acceleration Profile



Scenario #2 (with additional drag due to turn-rate)



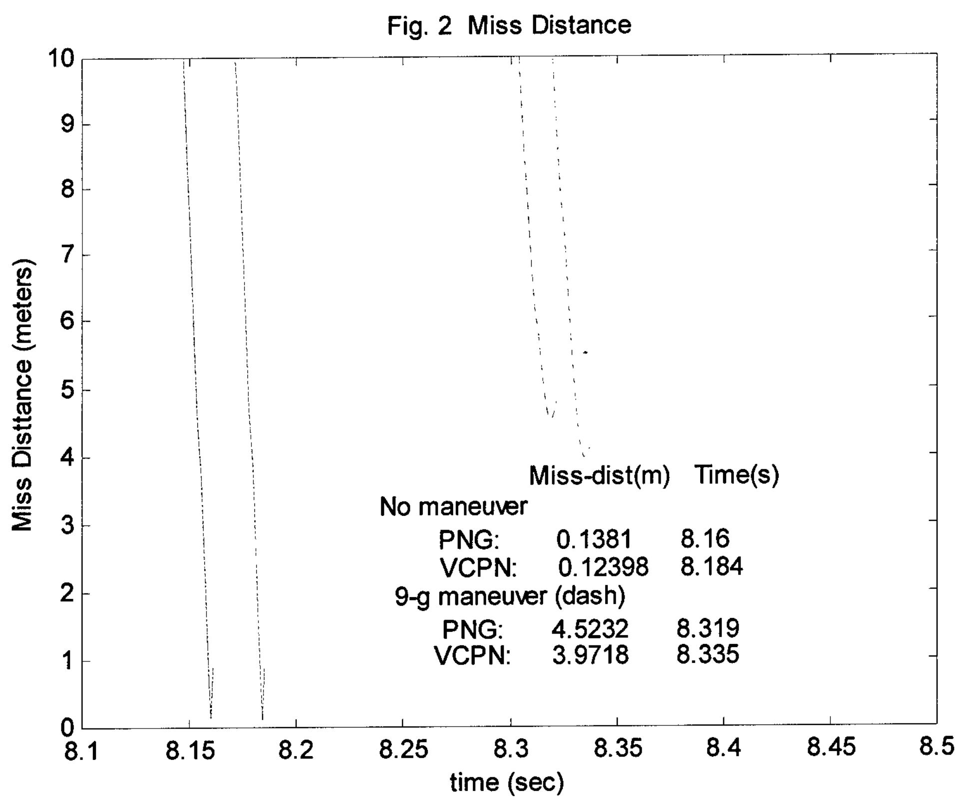


Fig. 3 Missile Velocity

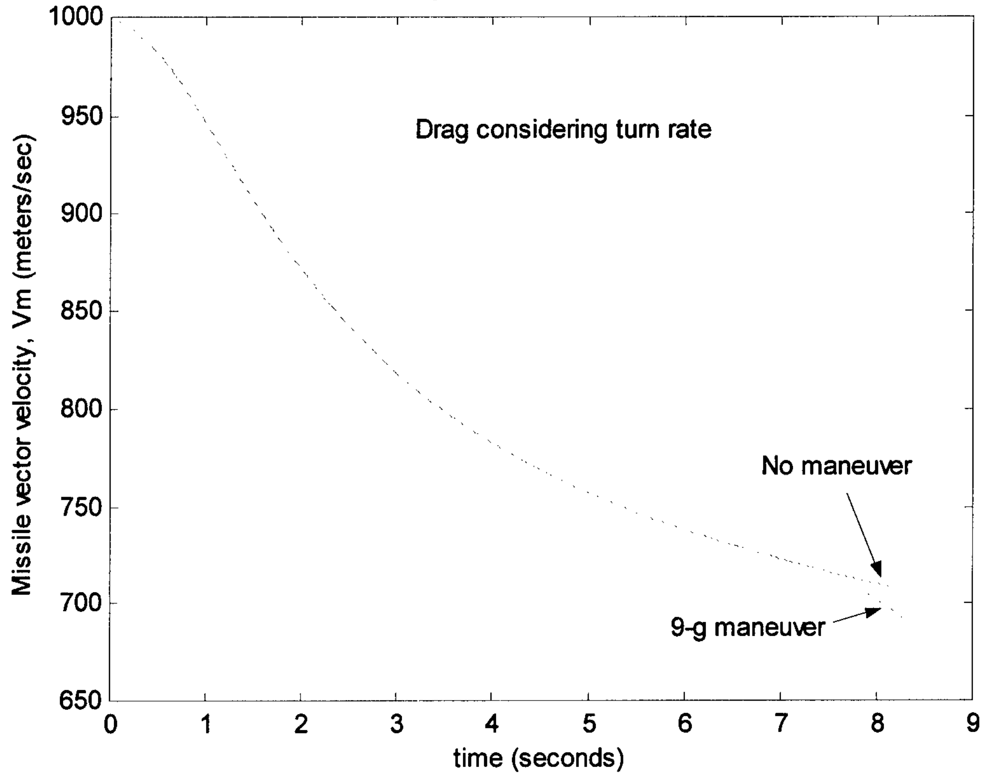
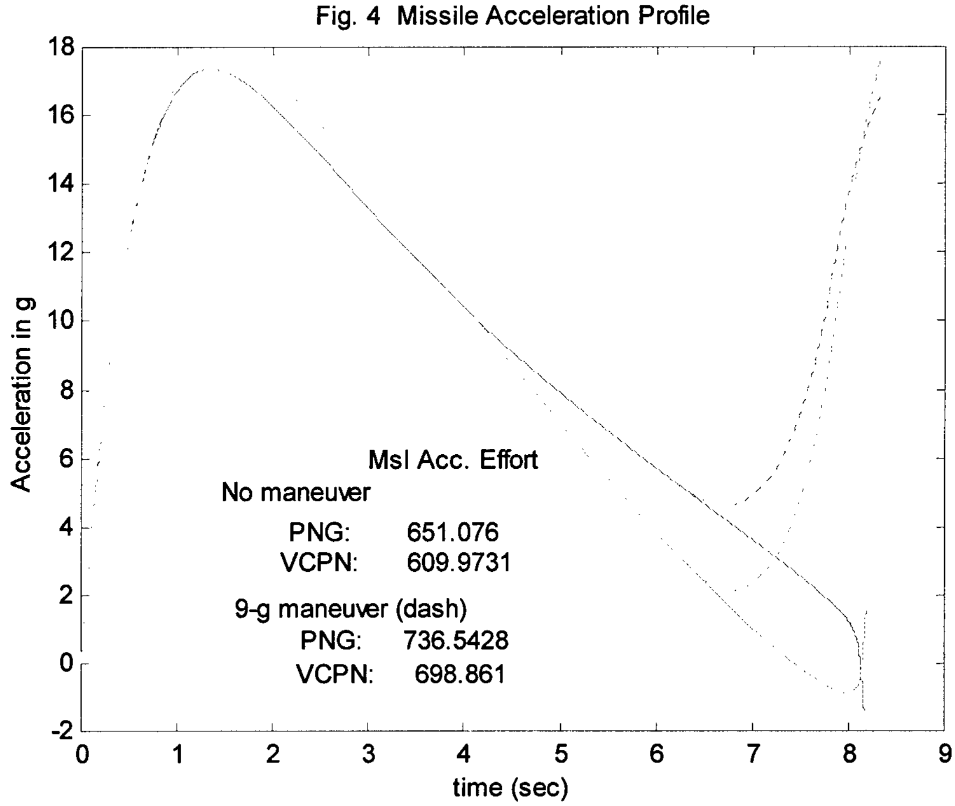
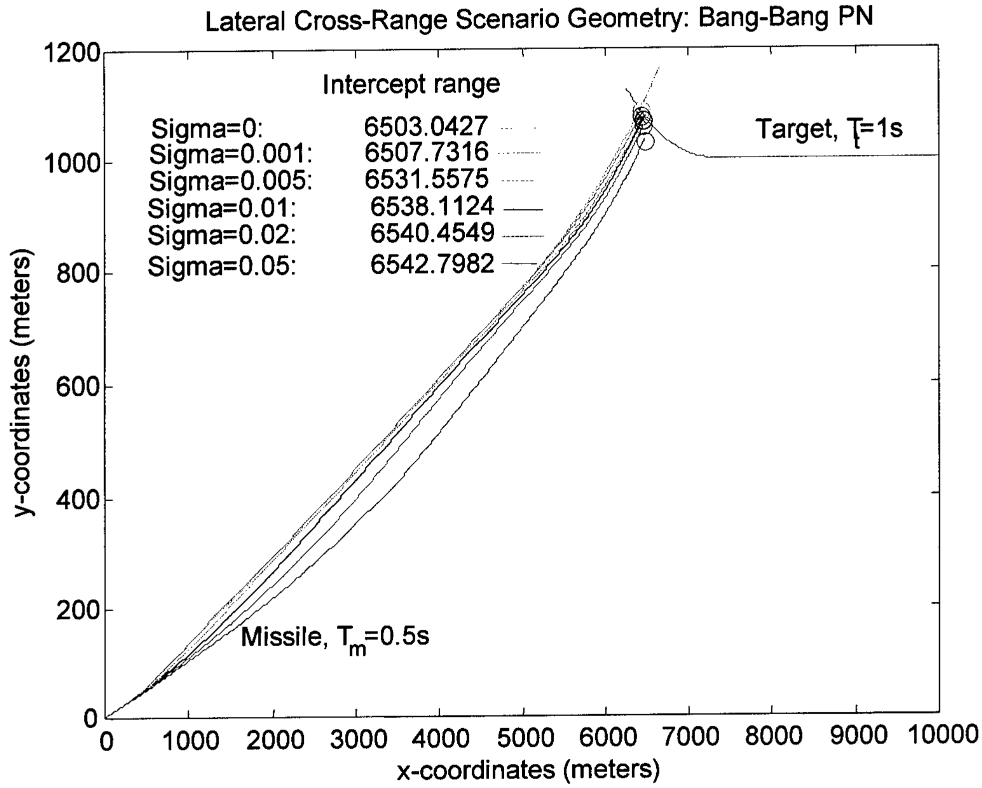


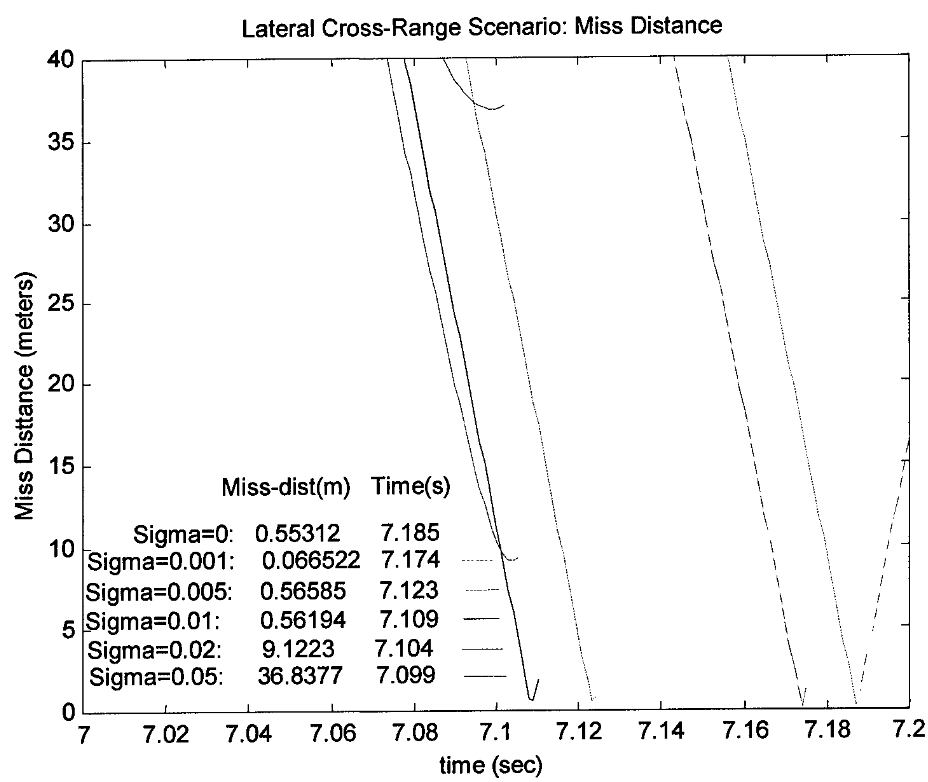
Fig. 4 Missile Acceleration Profile

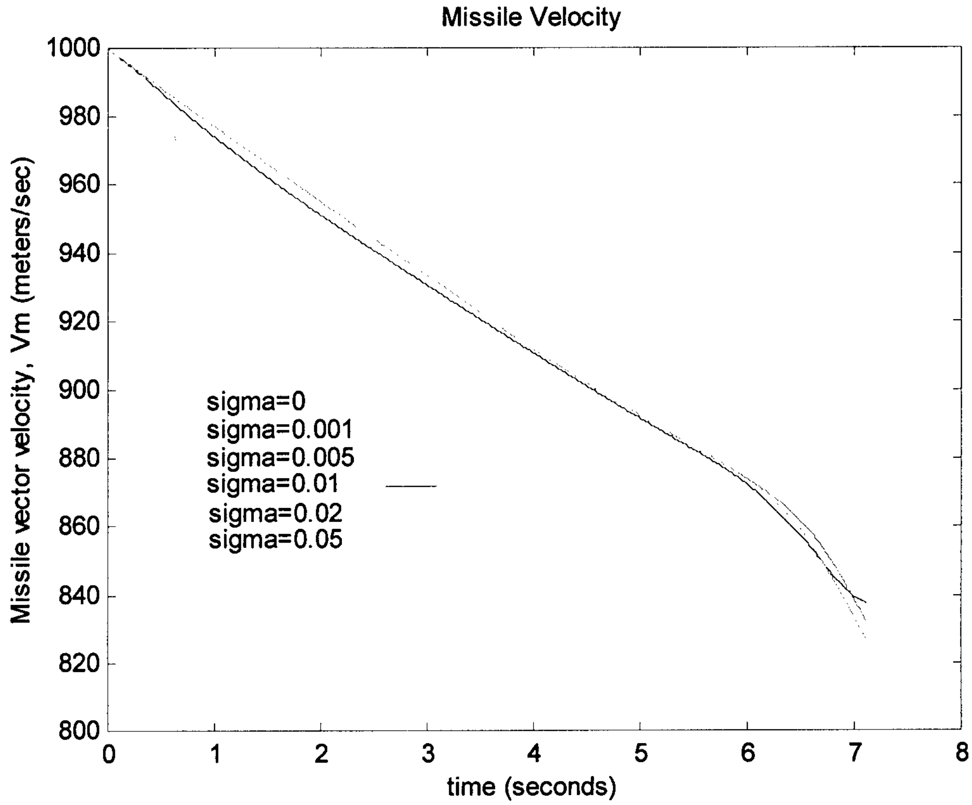


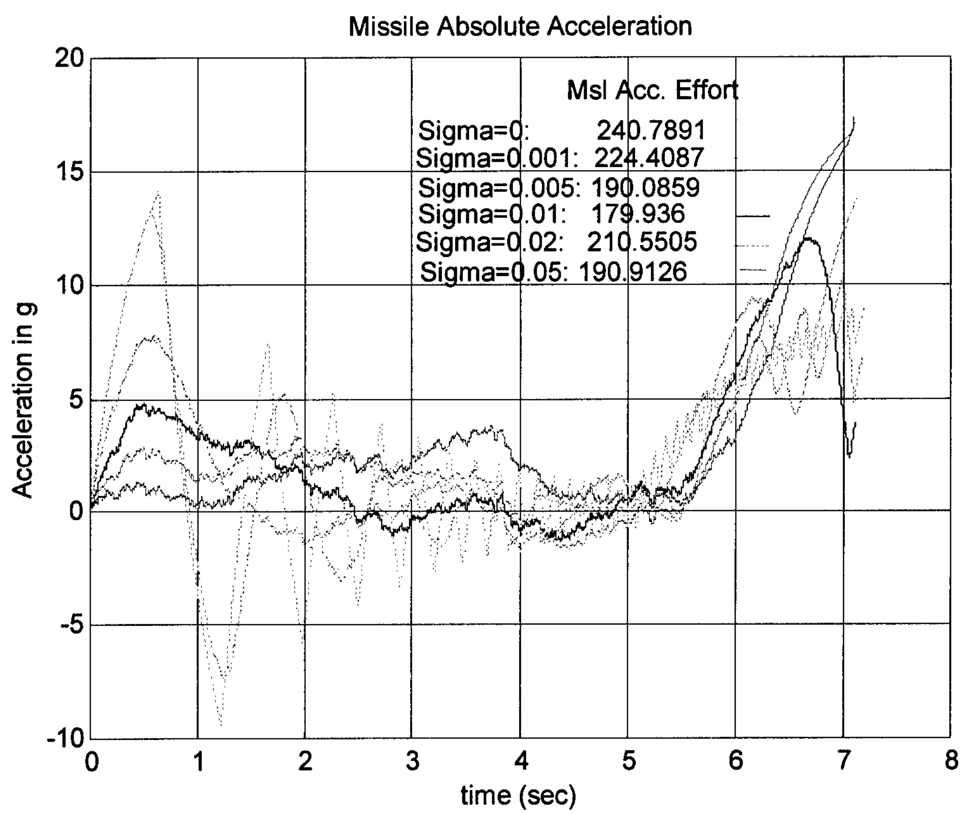
3. Bang-bang

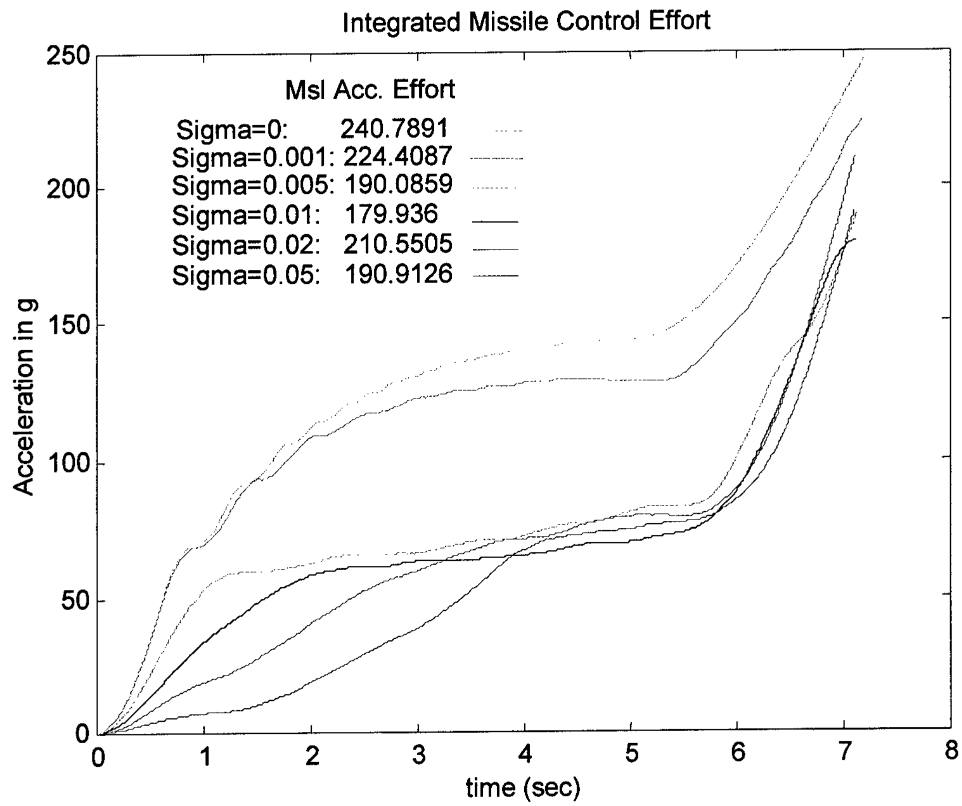
Scenario #1

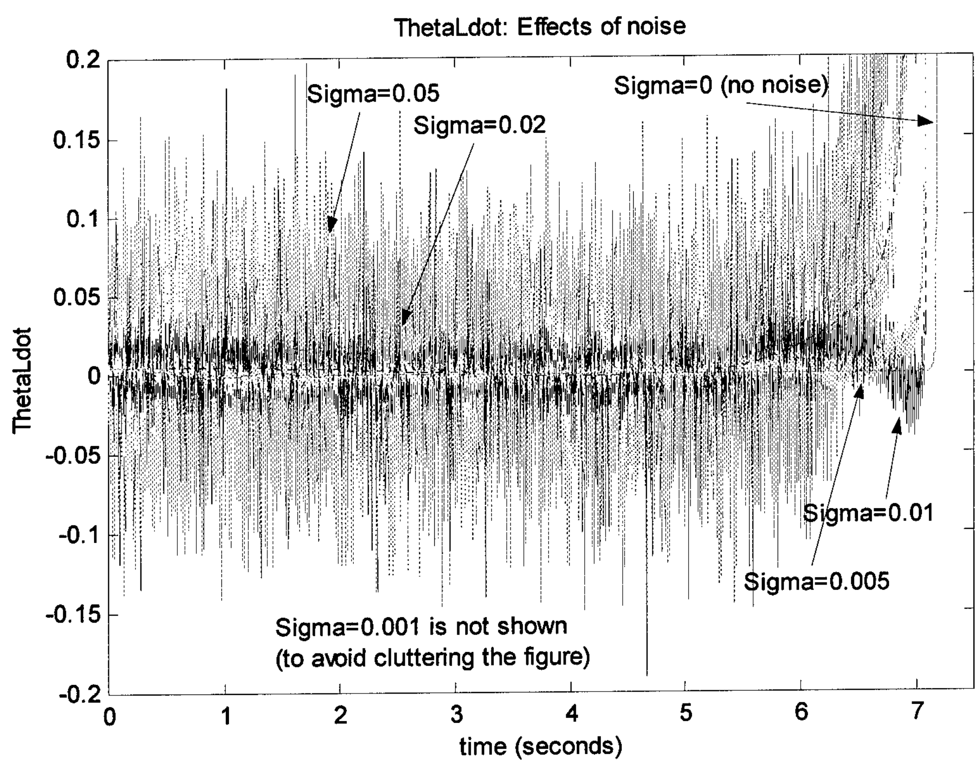


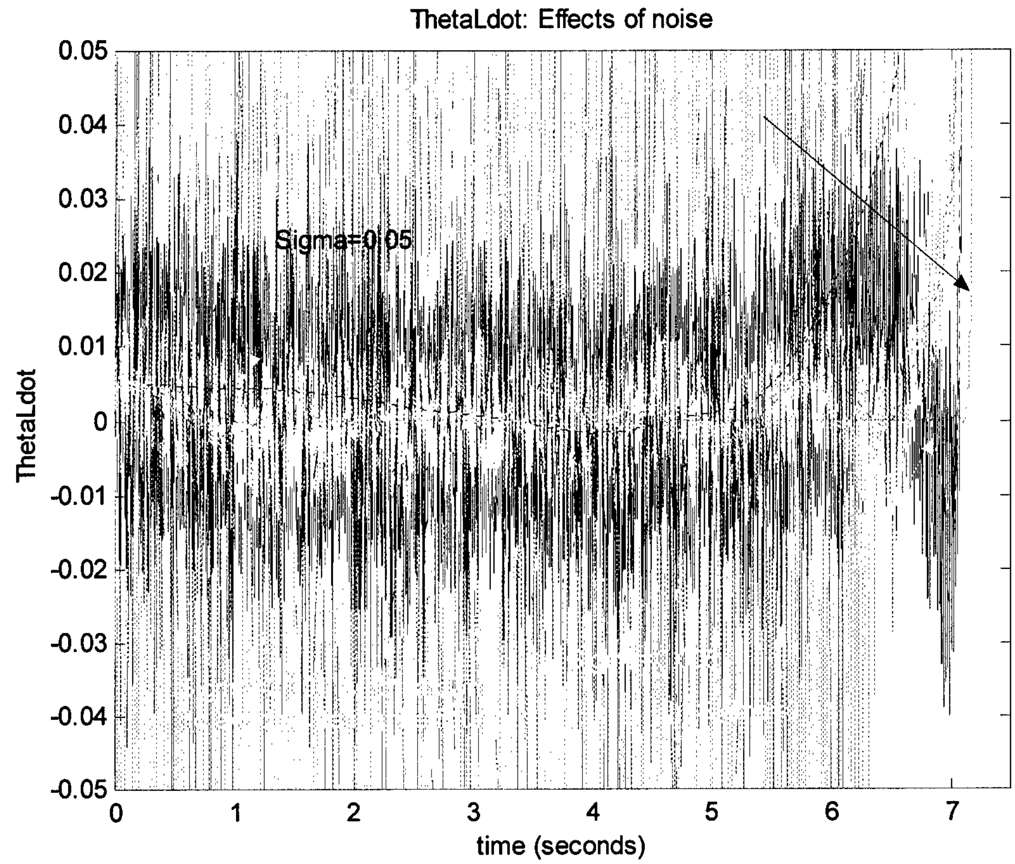


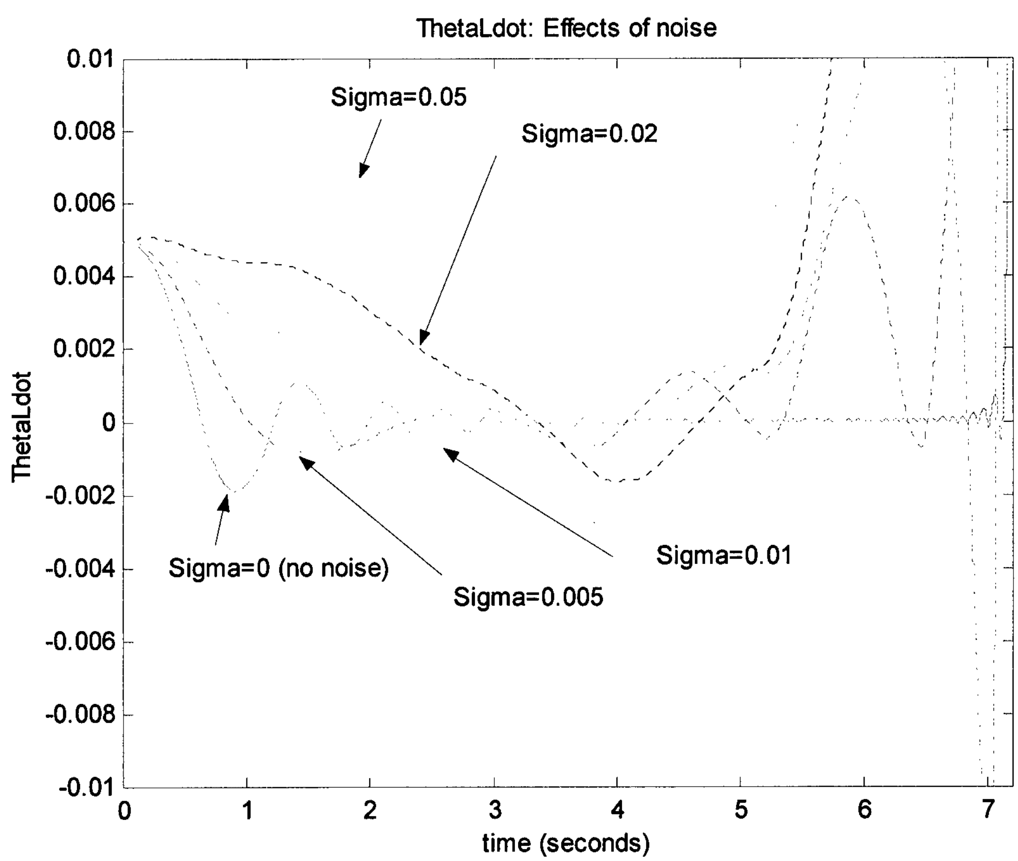

Missile Velocity
Missile vector velocity, Vm (meters/sec)
time (seconds)
sigma=0
sigma=0.001
sigma=0.005
sigma=0.01
sigma=0.02
sigma=0.05
1000
980
960
940
920
900
880
860
840
820
800
0
1
2
3
4
5
6
7
8


Missile Absolute Acceleration
Acceleration in g
time (sec)
Msl Acc. Effort
Sigma=0: 240.7891
Sigma=0.001: 224.4087
Sigma=0.005: 190.0859
Sigma=0.01: 179.936
Sigma=0.02: 210.5505
Sigma=0.05: 190.9126
20
15
10
5
0
-5
-10
0
1
2
3
4
5
6
7
8

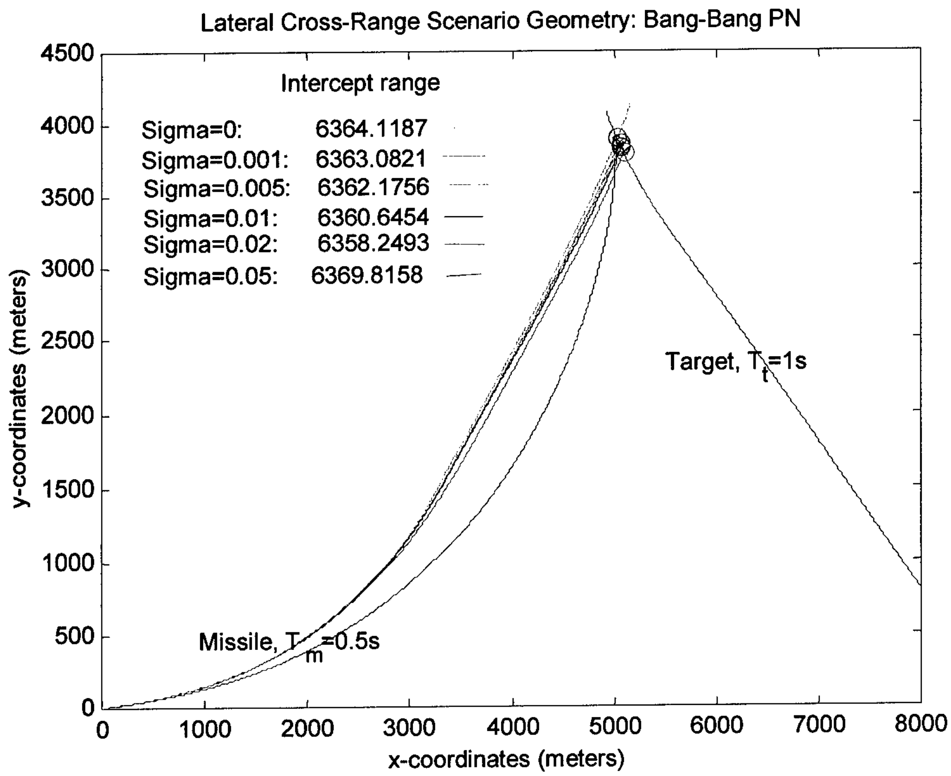

Integrated Missile Control Effort
Msl Acc. Effort
Sigma=0: 240.7891
Sigma=0.001: 224.4087
Sigma=0.005: 190.0859
Sigma=0.01: 179.936
Sigma=0.02: 210.5505
Sigma=0.05: 190.9126
Acceleration in g
time (sec)
0
50
100
150
200
250
1
2
3
4
5
6
7
8


ThetaLdot: Effects of noise
Sigma=0.05
Sigma=0.02
Sigma=0 (no noise)
Sigma=0.01
Sigma=0.005
Sigma=0.001 is not shown
(to avoid cluttering the figure)
ThetaLdot
time (seconds)
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
1
2
3
4
5
6
7

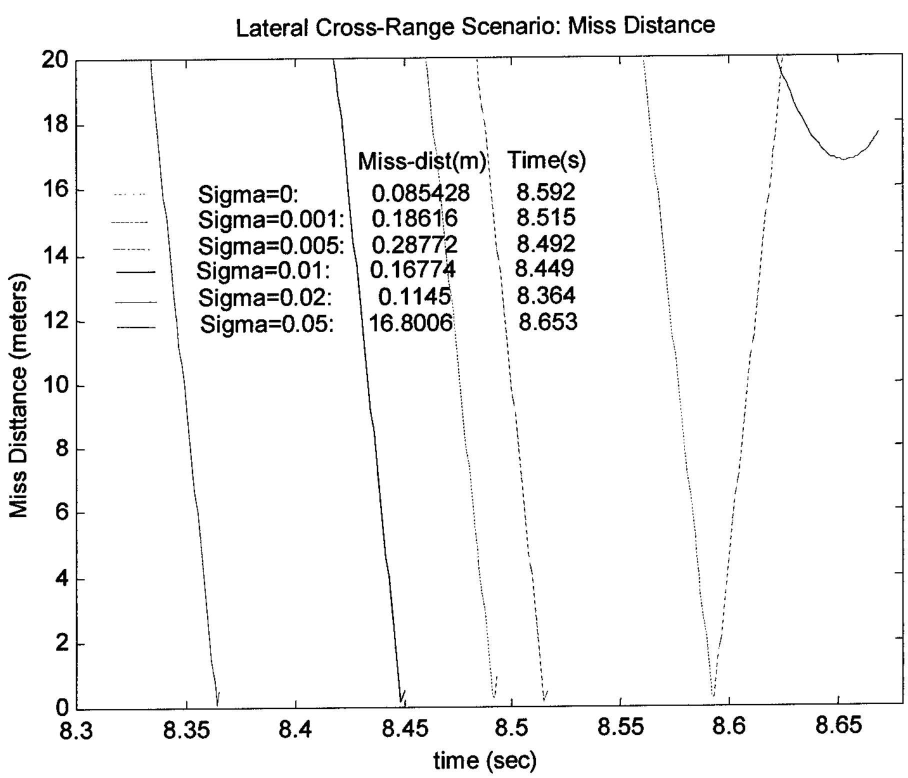

ThetaLdot: Effects of noise
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
ThetaLdot
Sigma=0.05
0
1
2
3
4
5
6
7
time (seconds)


ThetaLdot: Effects of noise
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
ThetaLdot
Sigma=0.05
Sigma=0.02
Sigma=0 (no noise)
Sigma=0.005
Sigma=0.01
0
1
2
3
4
5
6
7
time (seconds)

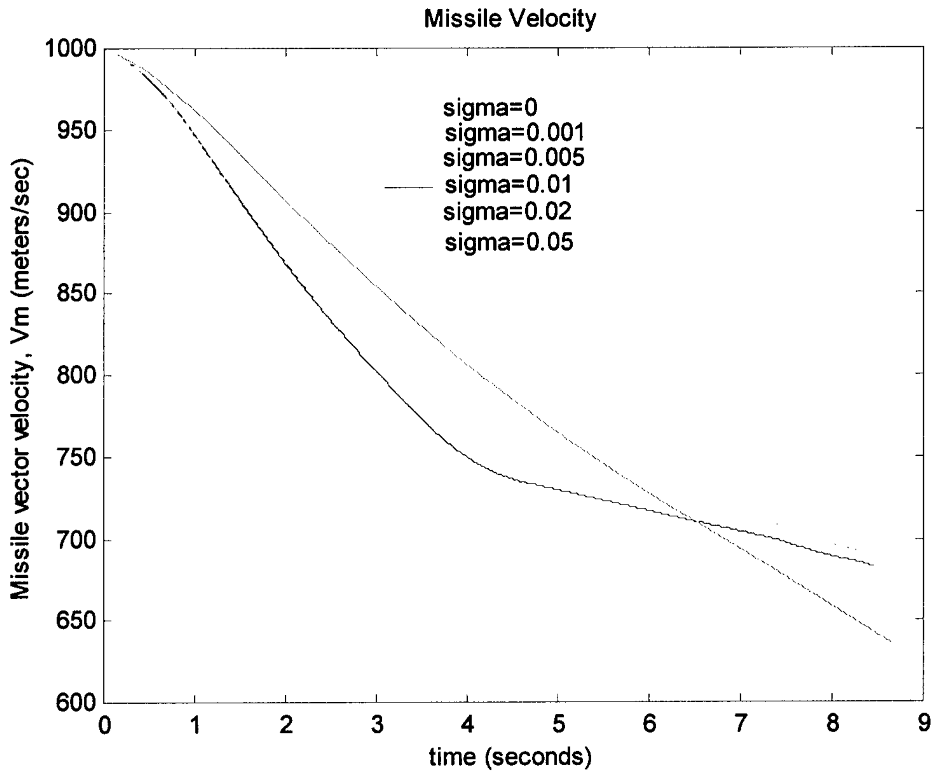
Scenario#2

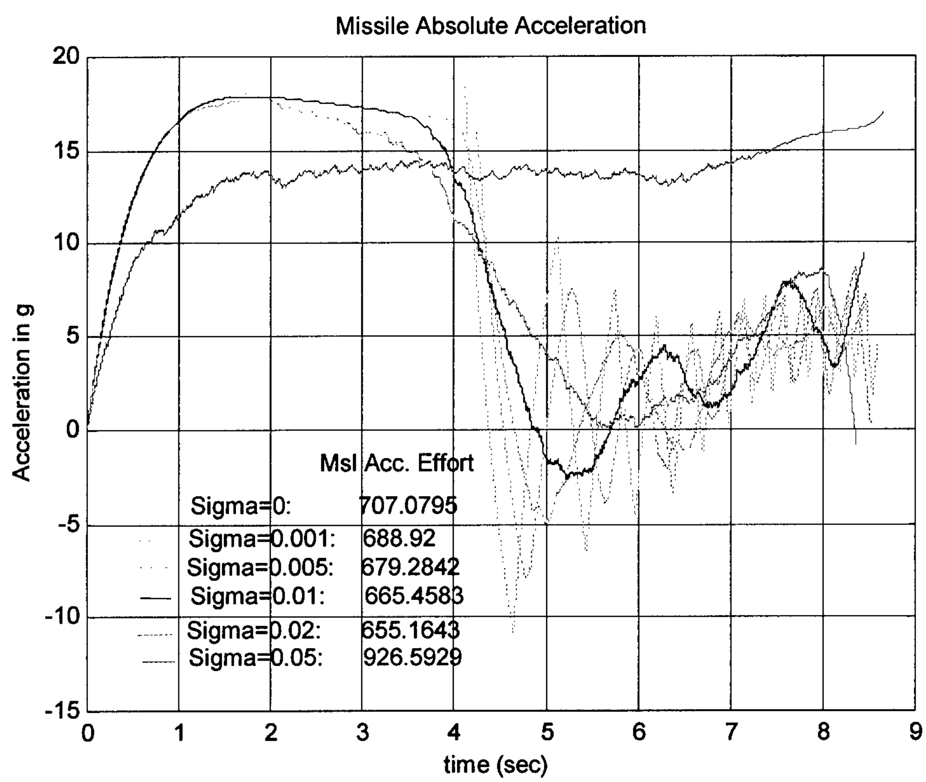


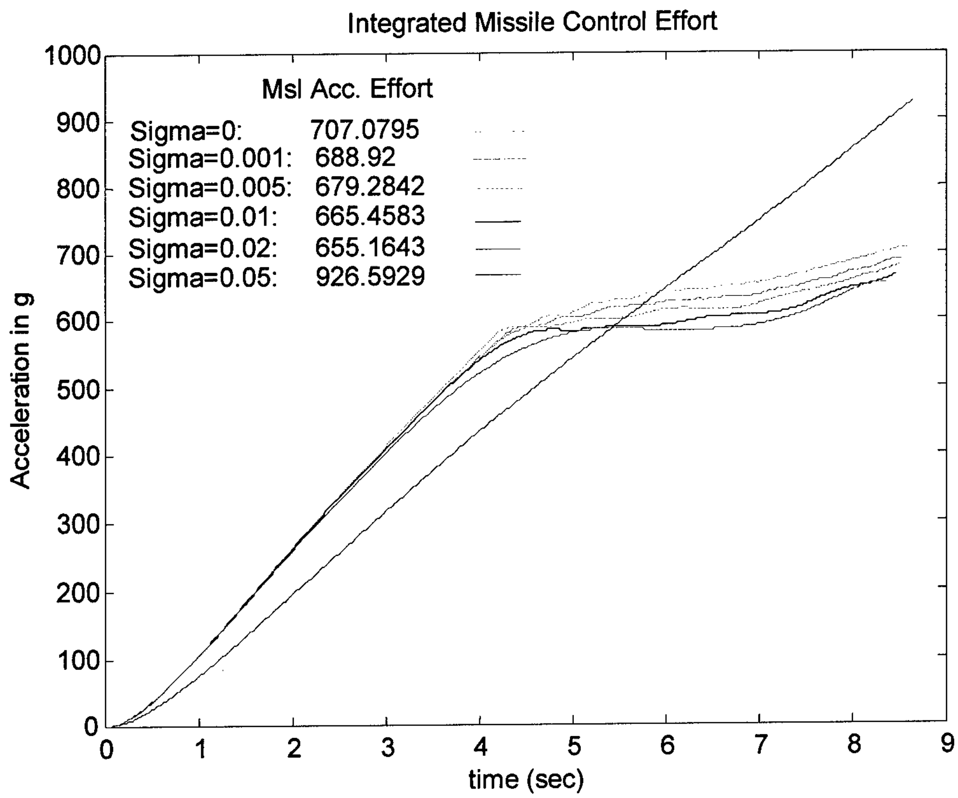
Lateral Cross-Range Scenario Geometry: Bang-Bang PN
Intercept range
Sigma=0: 6364.1187
Sigma=0.001: 6363.0821
Sigma=0.005: 6362.1756
Sigma=0.01: 6360.6454
Sigma=0.02: 6358.2493
Sigma=0.05: 6369.8158
Target, T_t=1s
Missile, T_m=0.5s
y-coordinates (meters)
x-coordinates (meters)

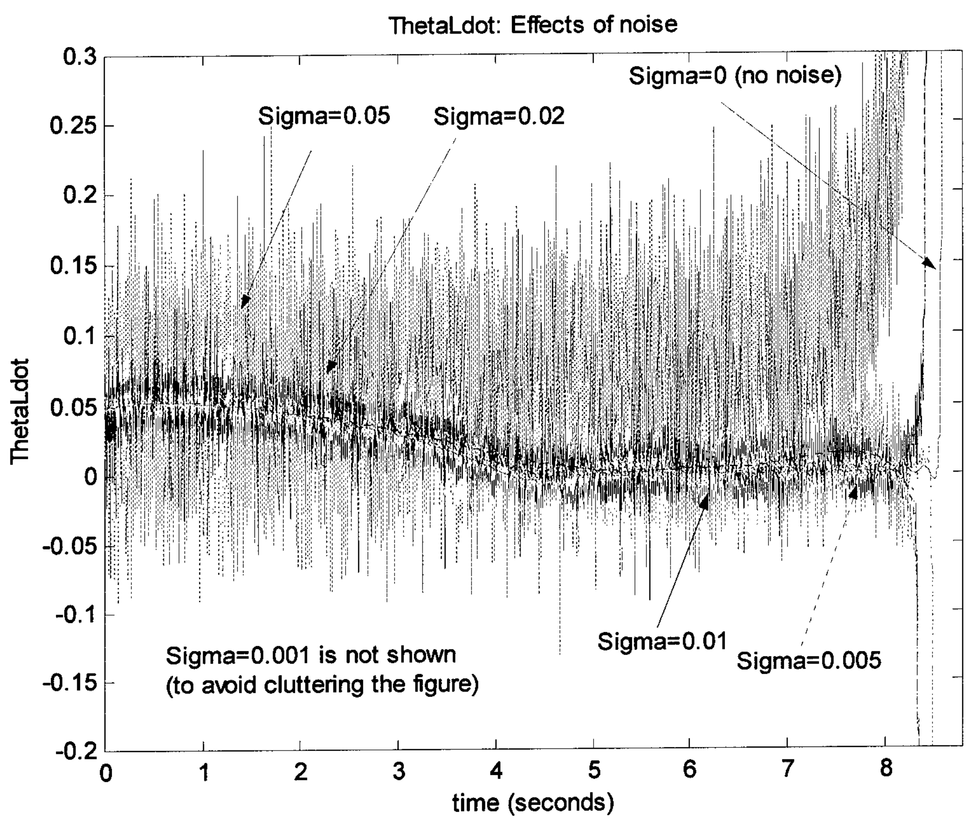


Lateral Cross-Range Scenario: Miss Distance
Miss-dist(m) Time(s)
Sigma=0: 0.085428 8.592
Sigma=0.001: 0.18616 8.515
Sigma=0.005: 0.28772 8.492
Sigma=0.01: 0.16774 8.449
Sigma=0.02: 0.1145 8.364
Sigma=0.05: 16.8006 8.653
Miss Disttance (meters)
time (sec)


Missile Velocity
sigma=0
sigma=0.001
sigma=0.005
sigma=0.01
sigma=0.02
sigma=0.05
Missile vector velocity, Vm (meters/sec)
time (seconds)


Missile Absolute Acceleration
Msl Acc. Effort
Sigma=0: 707.0795
Sigma=0.001: 688.92
Sigma=0.005: 679.2842
Sigma=0.01: 665.4583
Sigma=0.02: 655.1643
Sigma=0.05: 926.5929
Acceleration in g
time (sec)


Integrated Missile Control Effort
Msl Acc. Effort
Sigma=0: 707.0795
Sigma=0.001: 688.92
Sigma=0.005: 679.2842
Sigma=0.01: 665.4583
Sigma=0.02: 655.1643
Sigma=0.05: 926.5929
Acceleration in g
time (sec)
1000
900
800
700
600
500
400
300
200
100
0
1
2
3
4
5
6
7
8
9


ThetaLdot: Effects of noise
Sigma=0 (no noise)
Sigma=0.05
Sigma=0.02
Sigma=0.001 is not shown
(to avoid cluttering the figure)
Sigma=0.01
Sigma=0.005
ThetaLdot
time (seconds)
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
1
2
3
4
5
6
7
8

4. Bang Bang with Prop Nav at R<2km

Scenario#1: Target Maneuvers with 9 g's 2 seconds from intercept

Fig. 1 Flight Trajectory

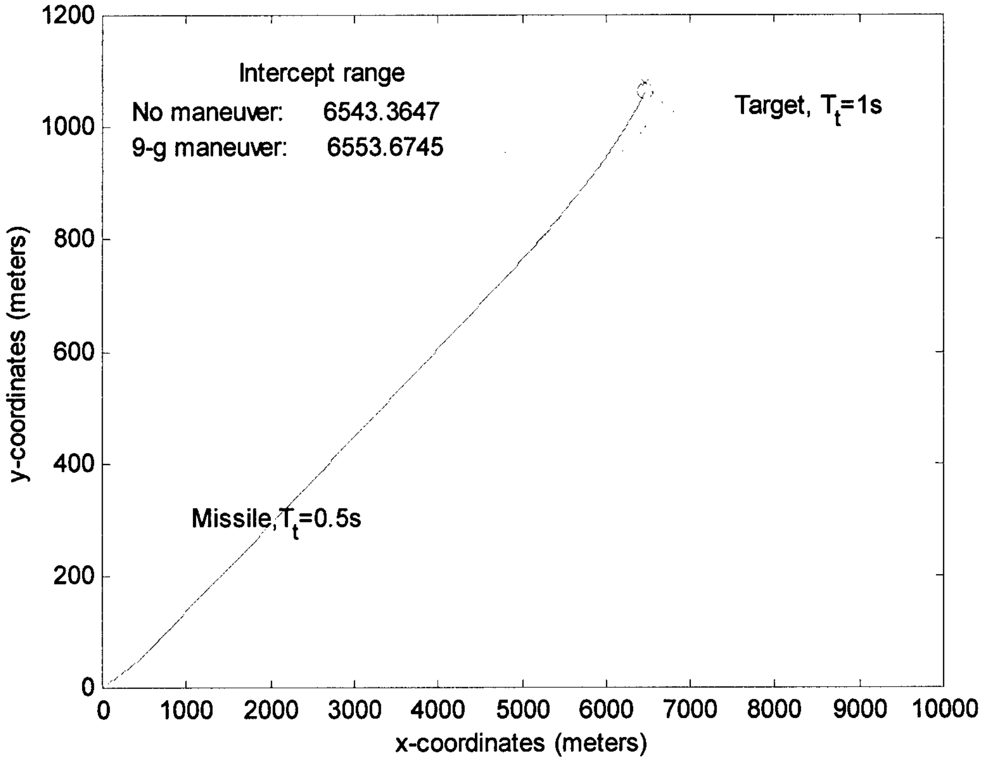


Fig. 2 Miss Distance

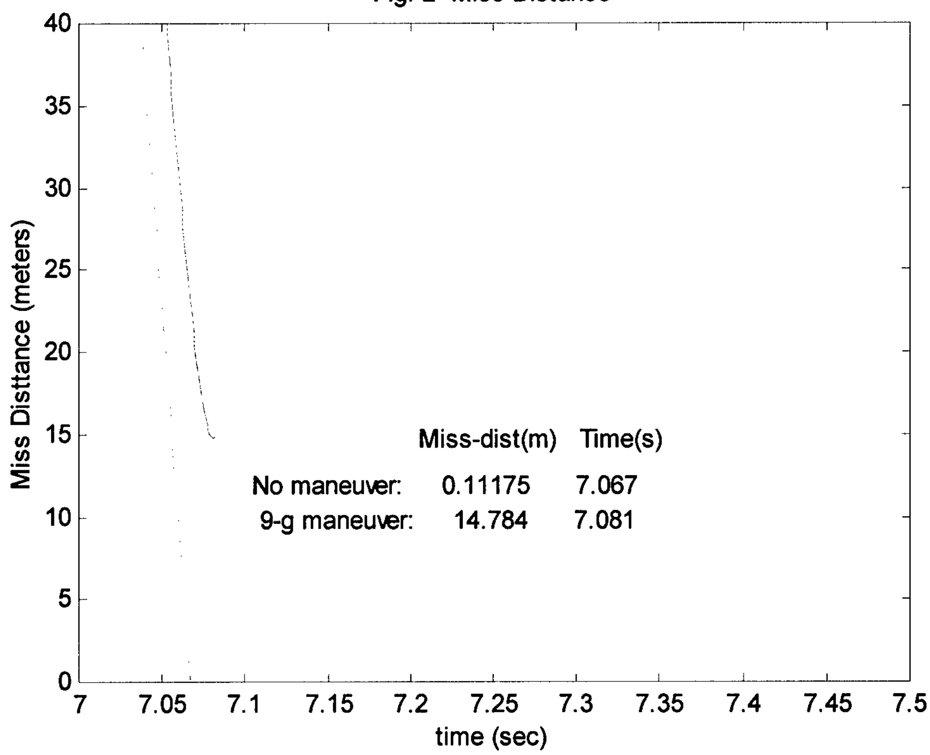
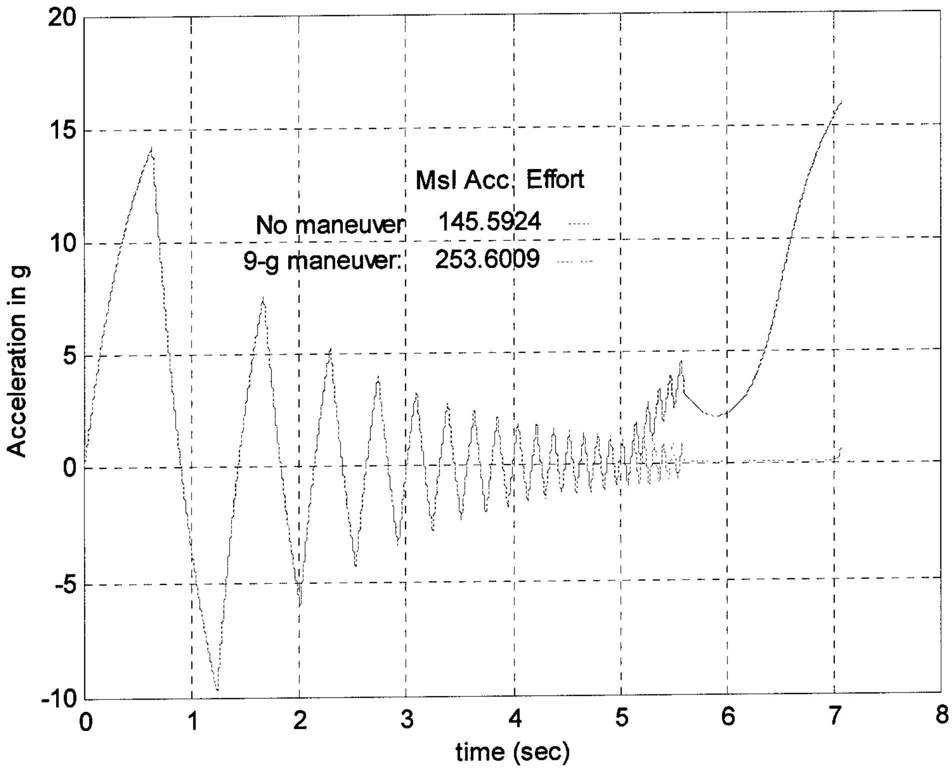


Fig. 4 Missile Acceleration Profile



Scenario#2:

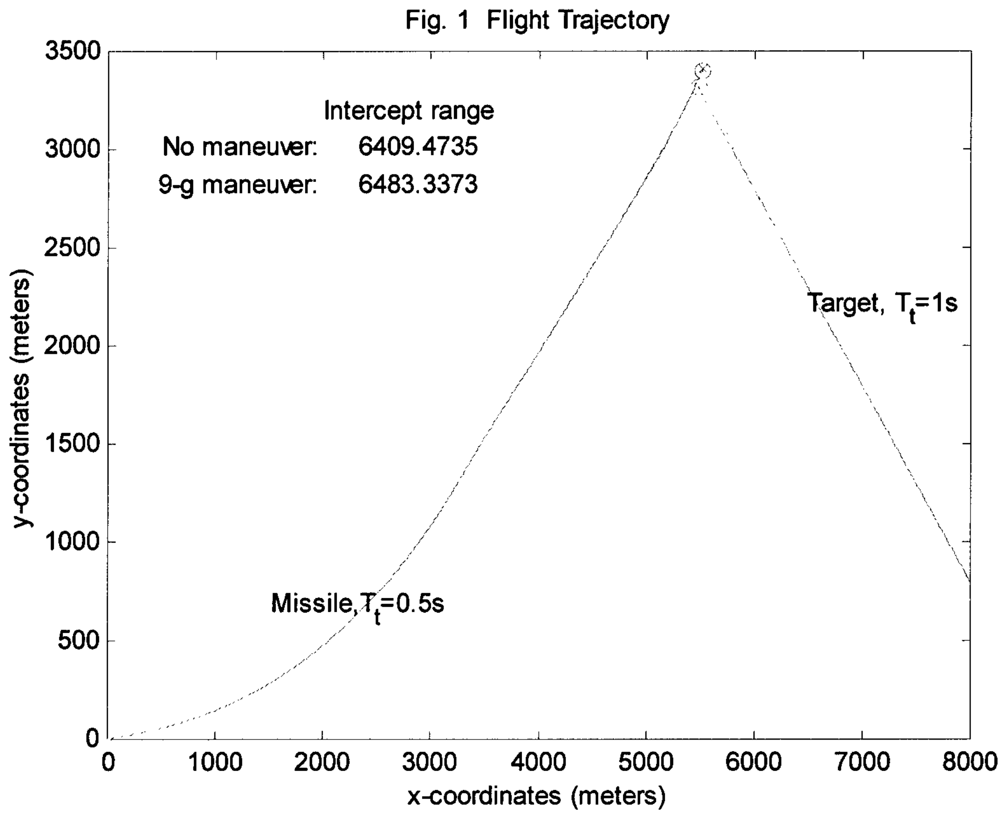


Fig. 1 Flight Trajectory
Intercept range
No maneuver: 6409.4735
9-g maneuver: 6483.3373
Target, T_t=1s
Missile, T_t=0.5s
y-coordinates (meters)
x-coordinates (meters)

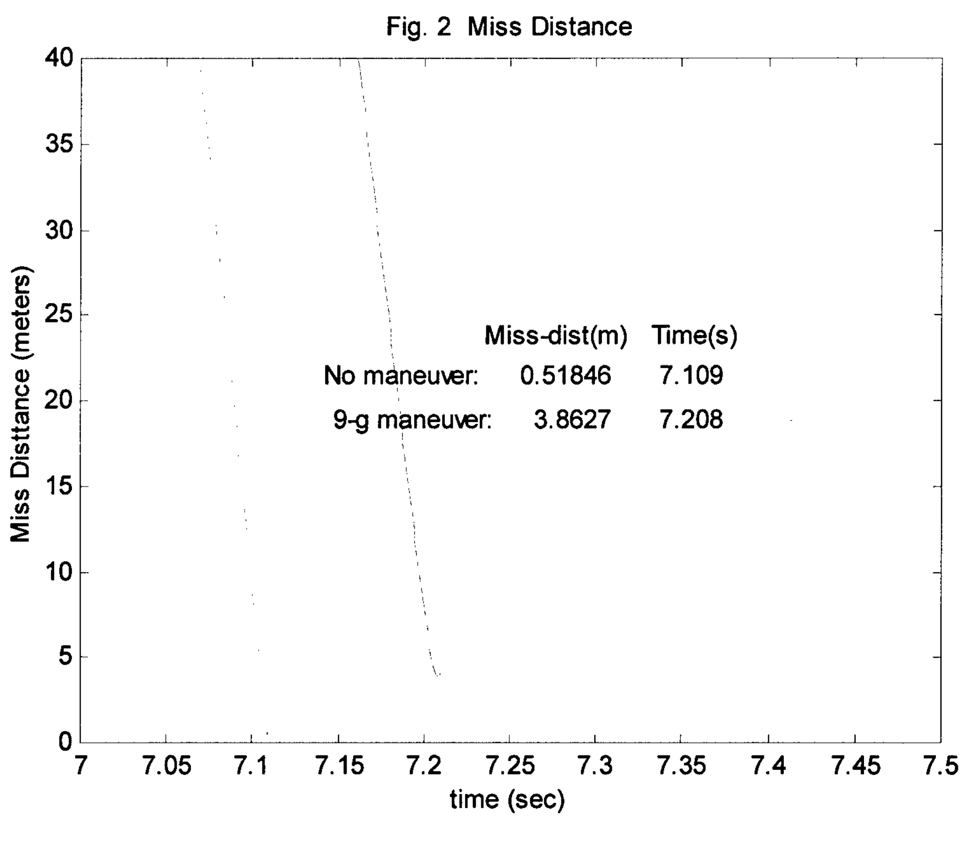
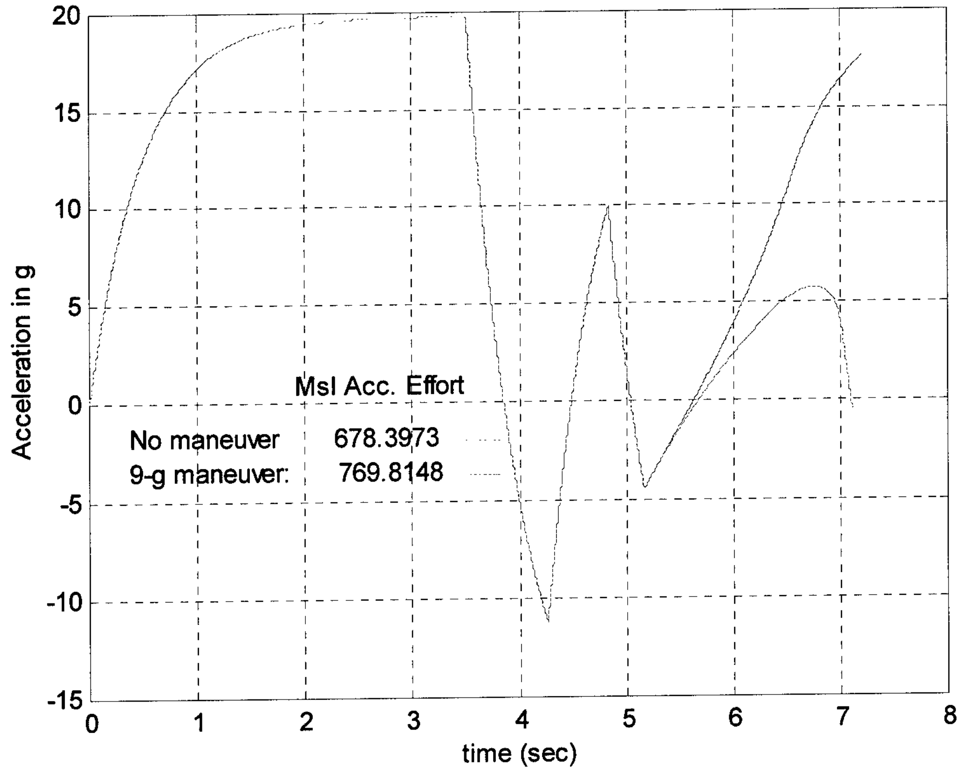


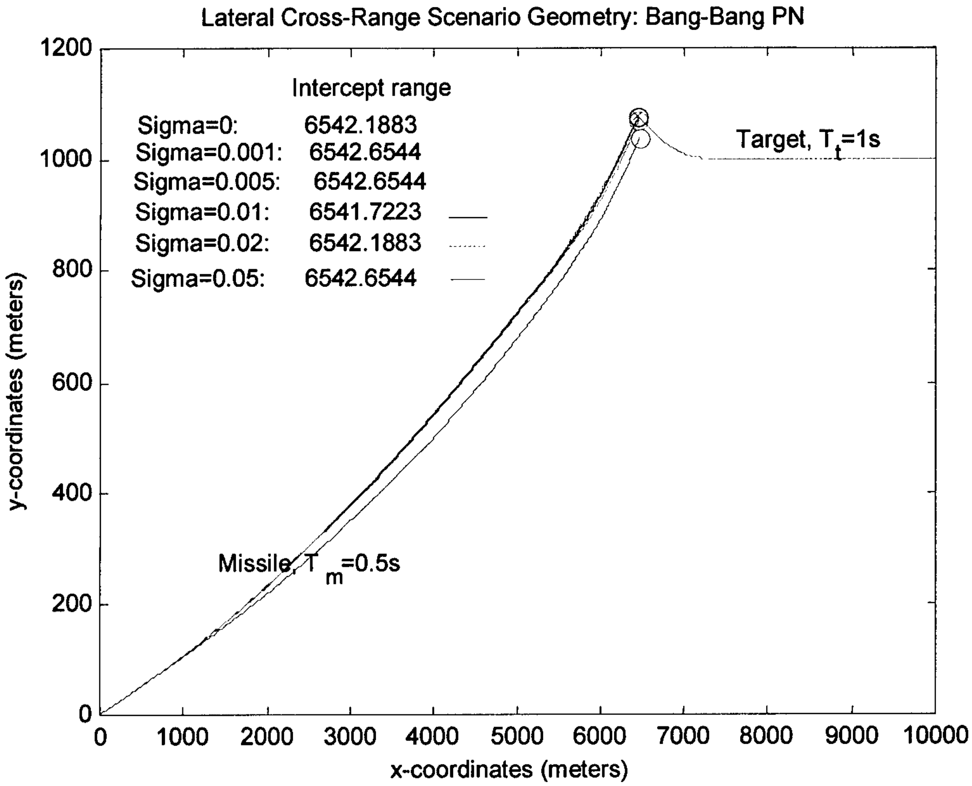
Fig. 2 Miss Distance
Miss-dist(m) Time(s)
No maneuver: 0.51846 7.109
9-g maneuver: 3.8627 7.208
Miss Disttance (meters)
time (sec)

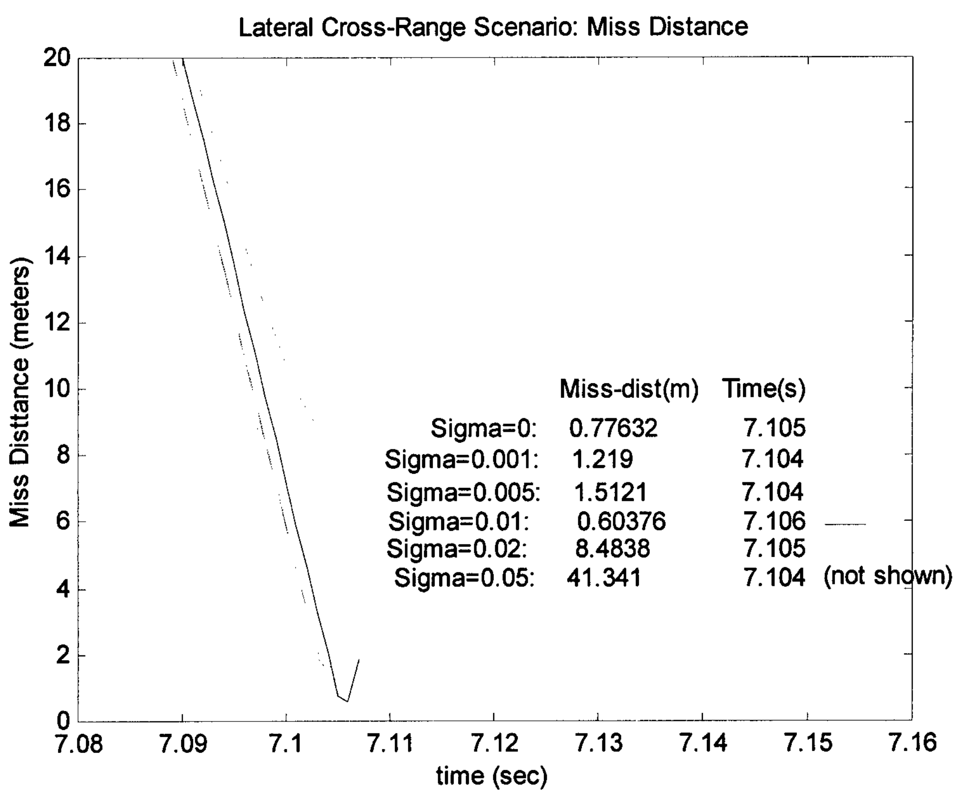
Fig. 4 Missile Acceleration Profile

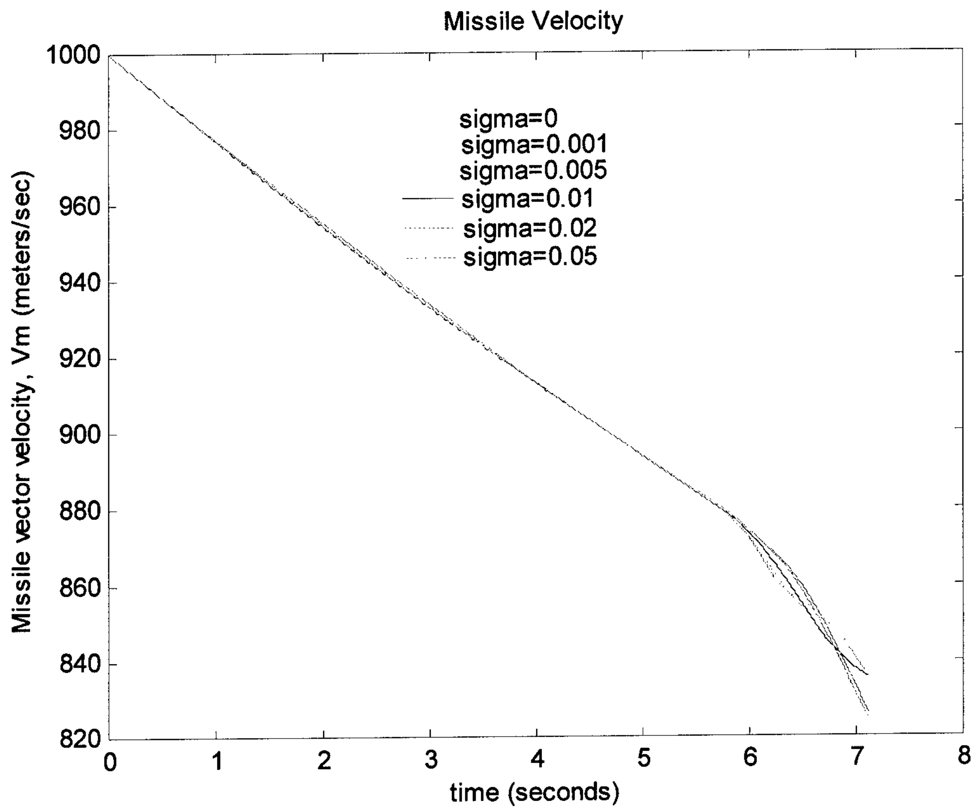


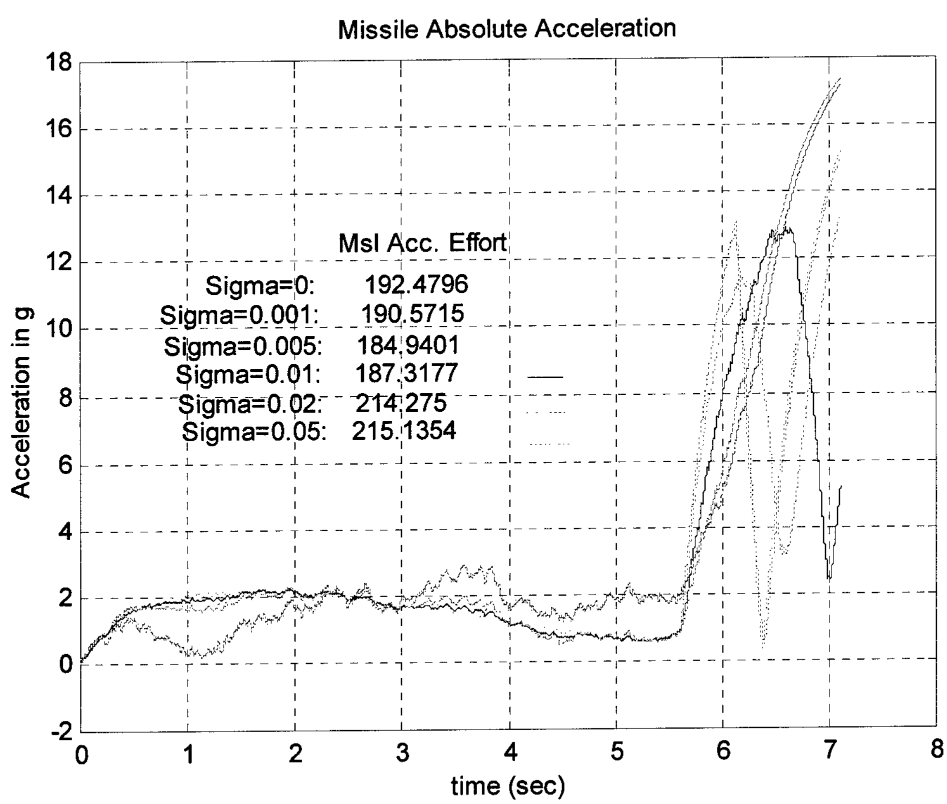
5. **Prop Nav with Bang Bang at R<2km**

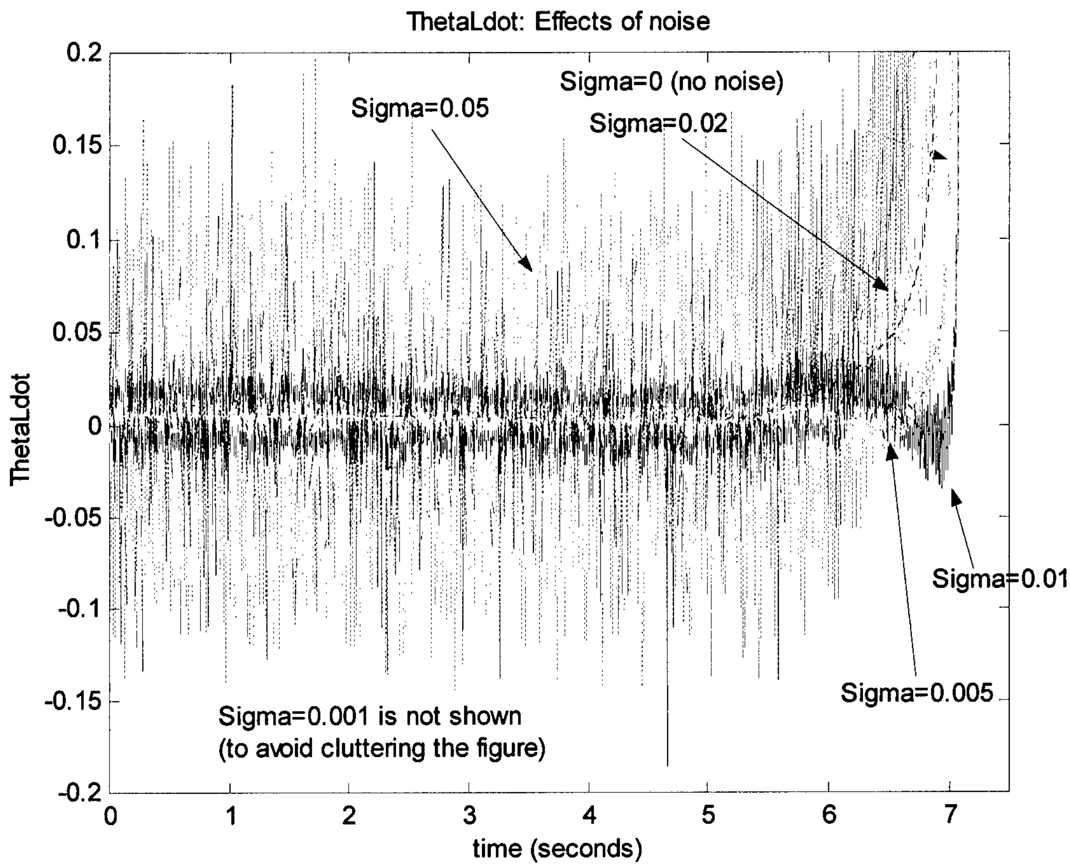
Scenario#1: Target Maneuvers with 9 g's 2 seconds from intercept

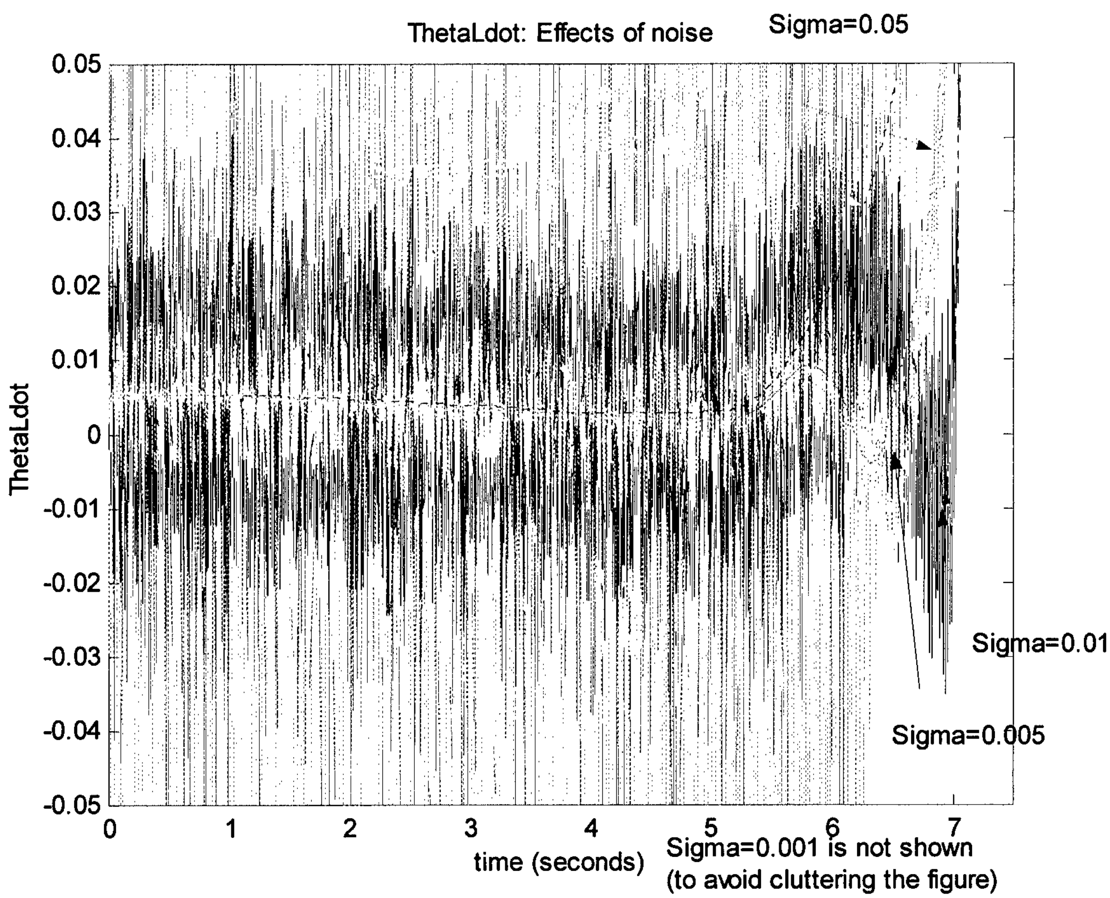




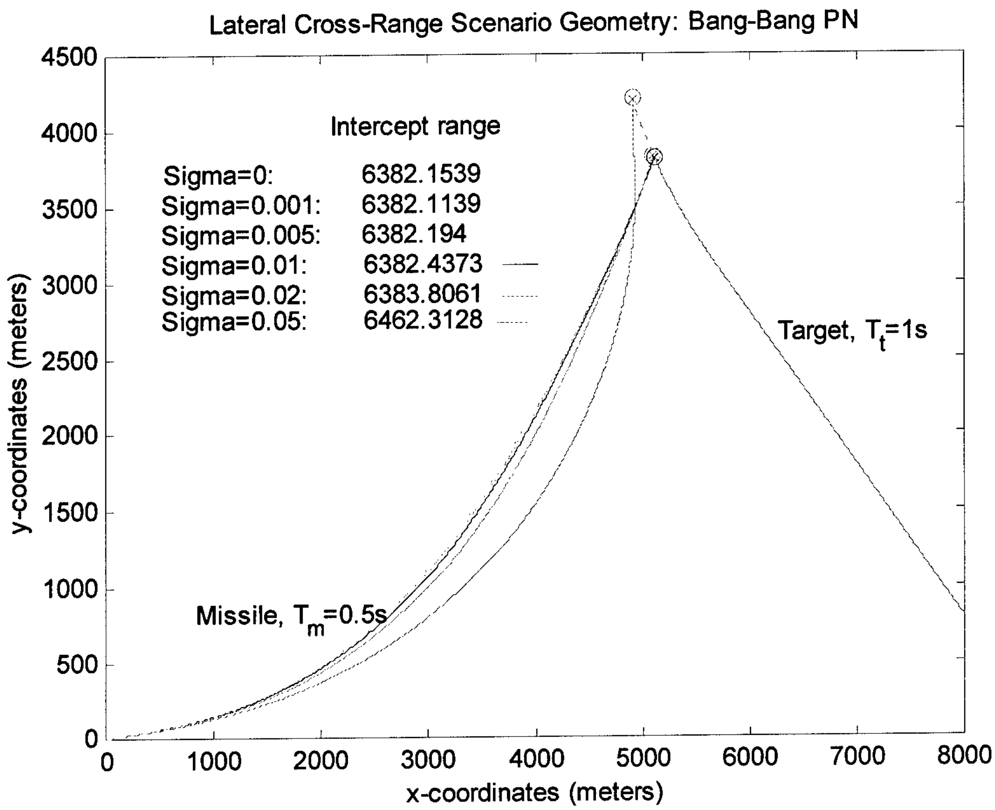

Missile Velocity
sigma=0
sigma=0.001
sigma=0.005
sigma=0.01
sigma=0.02
sigma=0.05
Missile vector velocity, Vm (meters/sec)
time (seconds)
1000
980
960
940
920
900
880
860
840
820
0
1
2
3
4
5
6
7
8


Missile Absolute Acceleration
Msl Acc. Effort
Sigma=0: 192.4796
Sigma=0.001: 190.5715
Sigma=0.005: 184.9401
Sigma=0.01: 187.3177
Sigma=0.02: 214.275
Sigma=0.05: 215.1354
Acceleration in g
time (sec)
18
16
14
12
10
8
6
4
2
0
-2
0
1
2
3
4
5
6
7
8

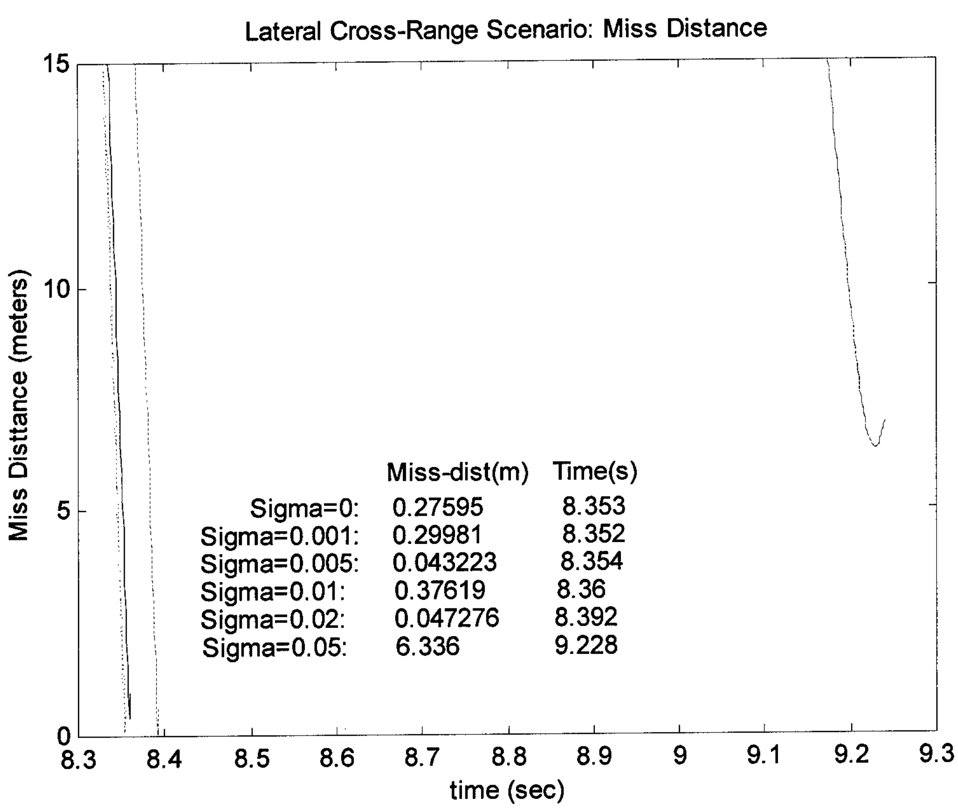

ThetaLdot: Effects of noise
Sigma=0 (no noise)
Sigma=0.05
Sigma=0.02
Sigma=0.01
Sigma=0.005
Sigma=0.001 is not shown
(to avoid cluttering the figure)
ThetaLdot
time (seconds)
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
1
2
3
4
5
6
7


ThetaLdot: Effects of noise
Sigma=0.05
Sigma=0.01
Sigma=0.005
Sigma=0.001 is not shown
(to avoid cluttering the figure)
ThetaLdot
time (seconds)
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0
1
2
3
4
5
6
7

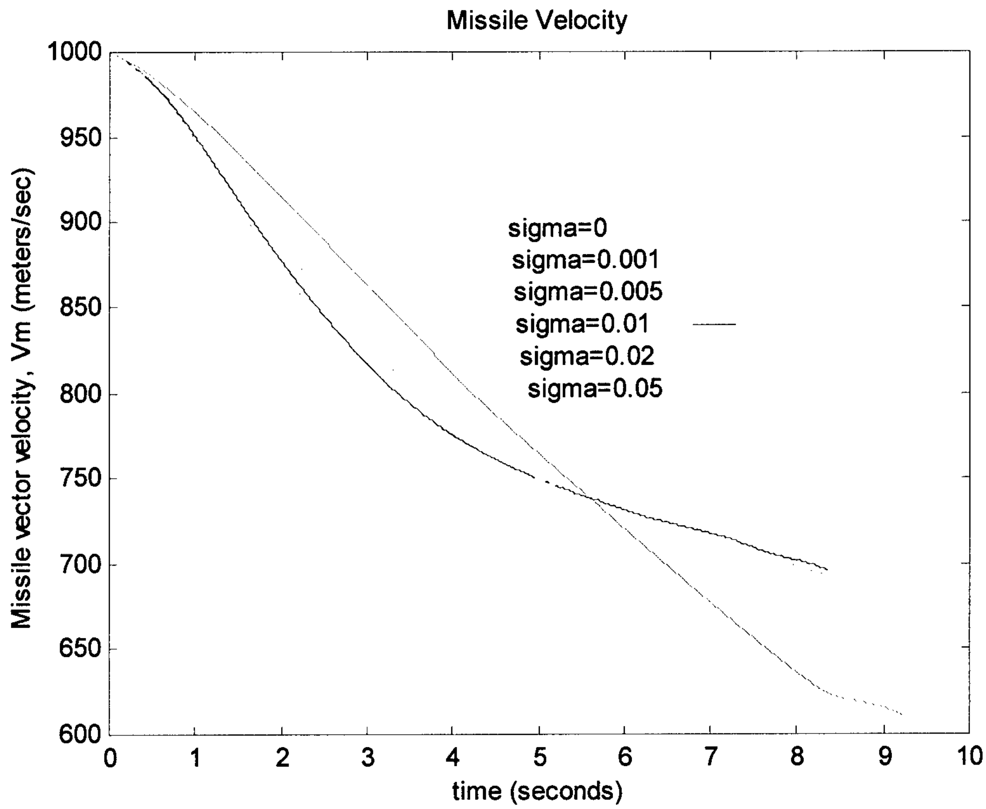
Scenario#2:



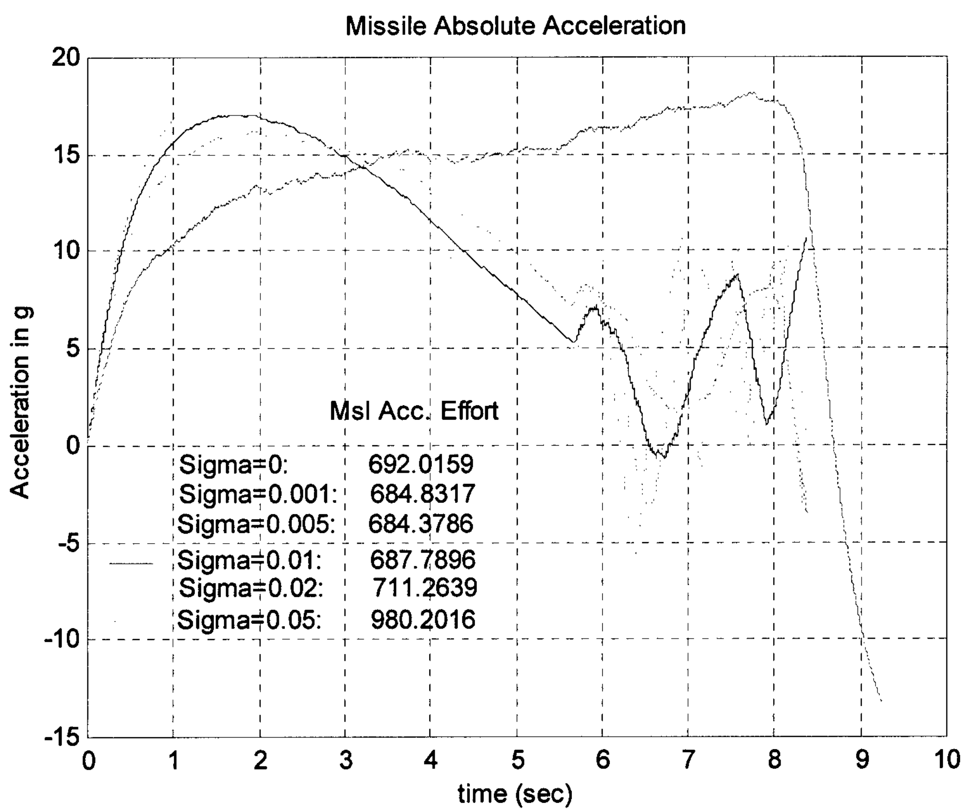
Lateral Cross-Range Scenario Geometry: Bang-Bang PN
Intercept range
Sigma=0: 6382.1539
Sigma=0.001: 6382.1139
Sigma=0.005: 6382.194
Sigma=0.01: 6382.4373
Sigma=0.02: 6383.8061
Sigma=0.05: 6462.3128
Target, T_t=1s
Missile, T_m=0.5s
y-coordinates (meters)
x-coordinates (meters)
0
500
1000
1500
2000
2500
3000
3500
4000
4500
0
1000
2000
3000
4000
5000
6000
7000
8000



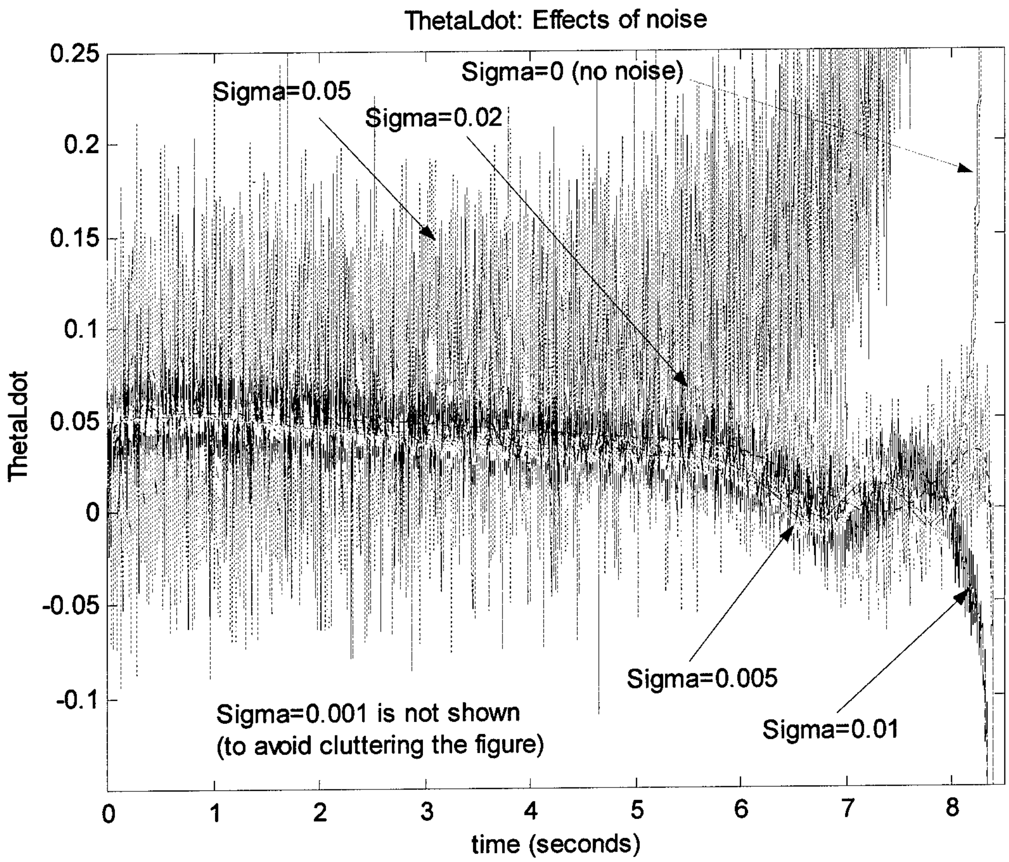
Lateral Cross-Range Scenario: Miss Distance
Miss-dist(m) Time(s)
Sigma=0: 0.27595 8.353
Sigma=0.001: 0.29981 8.352
Sigma=0.005: 0.043223 8.354
Sigma=0.01: 0.37619 8.36
Sigma=0.02: 0.047276 8.392
Sigma=0.05: 6.336 9.228
Miss Disttance (meters)
time (sec)
0
5
10
15
8.3
8.4
8.5
8.6
8.7
8.8
8.9
9
9.1
9.2
9.3

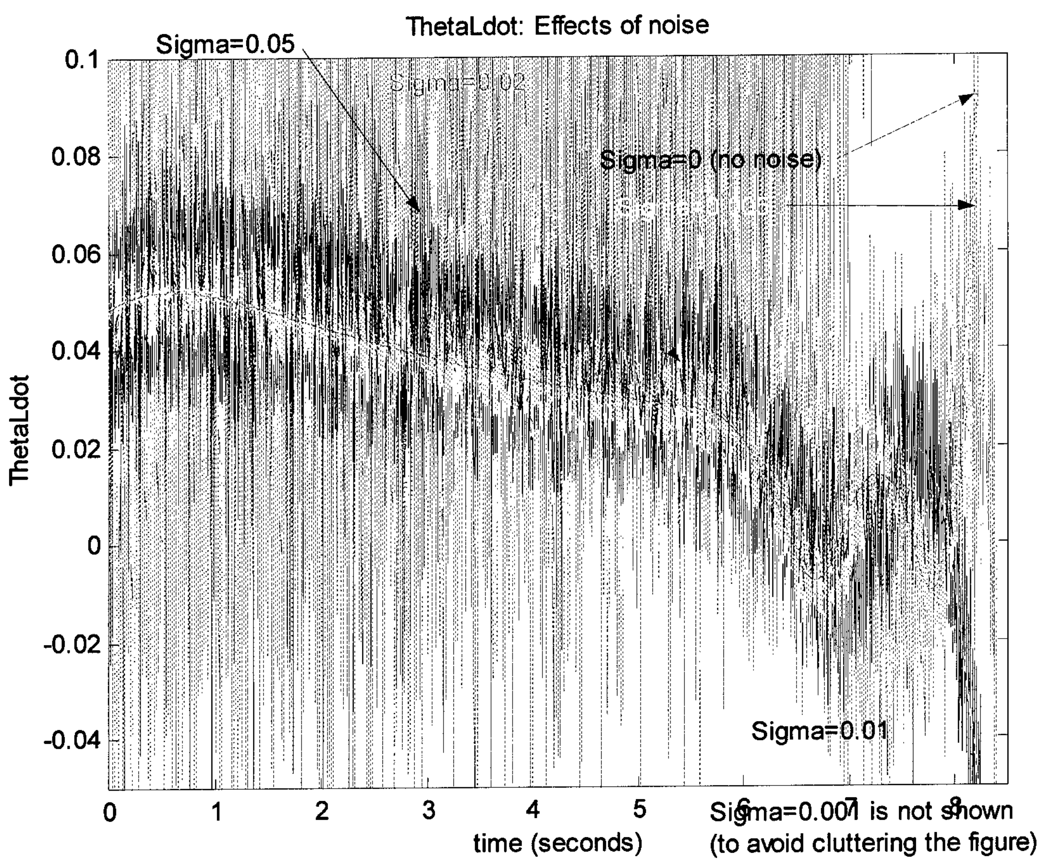


Missile Velocity
Missile vector velocity, Vm (meters/sec)
time (seconds)
sigma=0
sigma=0.001
sigma=0.005
sigma=0.01
sigma=0.02
sigma=0.05



Missile Absolute Acceleration
Acceleration in g
time (sec)
Msl Acc. Effort
Sigma=0: 692.0159
Sigma=0.001: 684.8317
Sigma=0.005: 684.3786
Sigma=0.01: 687.7896
Sigma=0.02: 711.2639
Sigma=0.05: 980.2016


ThetaLdot: Effects of noise
Sigma=0 (no noise)
Sigma=0.05
Sigma=0.02
Sigma=0.005
Sigma=0.01
Sigma=0.001 is not shown
(to avoid cluttering the figure)
ThetaLdot
time (seconds)
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
1
2
3
4
5
6
7
8


ThetaLdot: Effects of noise
Sigma=0.05
Sigma=0.02
Sigma=0 (no noise)
Sigma=0.01
Sigma=0.001 is not shown
(to avoid cluttering the figure)
ThetaLdot
time (seconds)
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
0
1
2
3
4
5
6
7
8

APPENDIX D. MATLAB® INFORMATION

MATLAB® and SIMULINK™ is a product of MathWorks, Inc., 24 Prime Way, Natick, Mass. 01760.

MATLAB® version 5.3 and SIMULINK™ were used throughout this study.

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4 Lin, Ching-Fang, *Modern Navigation, Guidance and Control Processing, Vol. II,* Prentice Hall, Englewood Cliffs, New Jersey, 1991.

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7 Becker, K., "Closed Form Solution of Pure Proportional Navigation," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 26, No.3, May 1990.

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9 Imado, F., Kurada, T., and Tahk M., "A New Missile Guidance Algorithm Against a Maneuvering Target," AIAA –98-4114, *Proceedings of Guidance Navigation and Control Conference, 1998.*

10 Ha, I., Hur, J., Ko, M., and Song, T., "Performance Analysis of PNG Laws for Randomly Maneuvering Target," AIAA –98-4114, *Proceedings of Guidance Navigation and Control Conference, 1998.*

11 Shinar, J., and Shima, T., "Guidance Law Evaluation in Highly Non-linear Scenarios- Comparison to Linear Analysis," AIAA –99-4065, *Proceedings of Guidance Navigation and Control Conference, 1999.*

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